Predatory Trading in a Rational Market*

Vincent Fardeau†

Frankfurt School of Finance and Management

January 2014

Abstract

This paper studies the feedback loop between liquidity and predatory trading. On one hand, predators exploit the market illiquidity to move prices and trigger a margin call on a rival trader (the prey)'s position. On the other hand, the mere anticipation of the prey's firesales by other market participants lowers the current price of the asset and changes the liquidity the prey has access to: her price impact decreases, while predators' increases. This makes predation cheaper from their viewpoint. The model predicts that predatory trading occurs in markets with low risk-bearing capacity, and shows that short-selling bans may be ineffective against predatory trading.

Keywords: Predatory trading, Firesales, Liquidity

JEL-Classification: C72, D43, G10

*This paper was previously circulated under the title: "Strategic Liquidity Provision and Predatory Trading" and "Predatory Trading with Smart Investor". I am greatly indebted to Dimitri Vayanos and Denis Gromb for their guidance and supervision. For helpful comments and discussions, I would also like to thank Miguel Antón, Ulf Axelson, Bernard Dumas, Mike Burkart, Mike Chernov, Amil Dasgupta, Daniel Ferreira, Fabian Garavito, Olivier Guéant, Harald Hau, Stéphane Guibaud, Yves Nosbusch, Joel Peress, Christopher Polk, Francesco Sangiorgi, Astrid Schornick, Yuki Sato, Michela Verardo, David Webb, Kathy Yuan, and seminar participants at the ECB, LSE, the Federal Reserve Board, INSEAD, the Spanish Economic Association Conference, the University of Melbourne, UNSW and the University of Vienna. All errors are my own.

†Department of Finance, Frankfurt School of Finance and Management, Sonnemannstr. 9-11, 60314 Frankfurt am Main, Germany. Email: v.fardeau@fs.de.

1 Introduction

Asset prices can at times exhibit sharp fluctuations. During these episodes, traders marking-to-market or relying on short-term funding (e.g. hedge funds, broker-dealers, investment banks) may become distressed and forced to sell assets at firesale prices. In some cases, it seems that the price movements causing these firesales can be exacerbated by deliberate strategies from traders seeking to profit from a rival's financial difficulties. There is evidence that such predatory trading occurred against LTCM in 1998 (Cai, 2009), and against several hedge funds during the recent financial crisis, in particular in the aftermath of Bear Stearns' and Lehmann Brothers' collapses[1].

Predatory trading strategies consist of two stages. First, some traders (predators) seek to cause or exacerbate price movements to decrease the marked-to-market value of a rival's portfolio. This tightens the prey's financial constraint, eventually leading to firesales. In a second stage, the predators gain by exploiting the firesale prices. Hence, on one hand, predatory trading relies critically on the market being imperfectly liquid: the predators must be able to move asset prices against the prey, and the prey's firesales must also affect prices. On the other hand, market liquidity should also depend on the possibility of predatory trading. Indeed, smart investors should anticipate that liquidity may temporarily dry up if a large trader liquidates her positions.[2] While predators manipulate the price to push their rival into distress, do smart investors absorb the predators' trades, thereby countering the predators' impact, or instead run for the exits, thus magnifying the liquidity dry-up?

[1]For instance, in March 2008, Focus Capital, a New York-based hedge fund specialized in mid-caps, was forced to close in the aftermath of Bear Stearns' collapse. The *Financial Times* wrote: "In a letter to investors, the founders of Focus, Tim O'Brien and Philippe Bubb, said it had been hit by 'violent short-selling by other market participants', which accelerated when rumours that it was in trouble circulated." (J. Mackintosh, FT, 4 March 2008). Similarly, in October 2008, the *Financial Times* wrote: "Hedge funds prey on rivals (...) the increasingly cannibalistic activity stems from a wave of redemptions hitting hedge funds " (H. Sender, FT, Oct 2008). See Brunnermeier and Pedersen (2005) for additional anecdotal evidence. Cai's (2009) paper documents dealers' predatory behaviour against LTCM in 1998, using a unique dataset of audit trail transactions.

[2]The financial constraints of a large trader may be known to other market participants. For instance, broker-dealers or lenders have information about the positions and balance sheet of large traders. Regulatory constraints sometimes impose to reveal positions, and although traders' identities may be concealed, market participants can often infer the positions of others from this information. For instance, Amaranth's positions in the natural gas market became known to other traders, who observed from the exchanges data that a single market participant had accumulated very large positions in the futures market (Levin and Coleman, 2007). More recently, hedge funds noticed the large positions accumulated by a trader at JP Morgan's Chief Investment Office (nicknamed the London whale) in the CDS markets and drew the attention of the financial press to the issue. The need to unwind the trades in this context eventually led to losses of about $6bn for JP Morgan.

Existing theories of predatory trading (e.g. Brunnermeier and Pedersen, 2005, Attari, Mello and Ruckes, 2005) are largely silent on this issue, because they assume that the predators and the prey trade with a competitive fringe of long-term value investors, whose demand is fixed.[3] This implies that these investors are less-than-fully rational in that they disregard future price movements. Without this assumption, it is not clear to which extent the results of these papers would remain or be qualified. In this paper, I show that predatory trading may occur even in the presence of smart investors who understand that the asset can be artificially and temporarily undervalued due to predatory trading-induced price pressure. Since holding the asset is risky, smart investors' willingness to "lean against the wind" and absorb predators' price pressure is limited by their risk-bearing capacity. Hence even if the mis-valuation is expected to disappear in the future, smart investors are unwilling to take unbounded positions ex-ante because it would expose them to too much risk. In fact, I show that smart investors' reactions to predatory trading may actually reduce the cost of predation for predators.[4]

When the competitive fringe of the market is made of smart investors, the predators' and the prey's price impacts, and more generally market liquidity, not only *affect*, but also *are affected by* the possibility of predatory trading. This two-way relationship can generate self-fulfilling liquidity dry-ups and make predatory trading cheaper for predators. I show that when smart investors expect the prey to fail in the future, current prices adjust to reflect the fact that the prey's firesales will lower the willingness of other market participants to hold the asset. Further, when a firesale is expected to occur, price impact becomes trader-specific and becomes an increasing function of a large trader's financial strength - or at least the smart investors' perception of it. That is, the prey's trades move prices less than opposite trades by predators. This reduces the prey's ability to resist predatory trading by supporting prices to avoid reporting a low marked-to-market wealth. Hence the mere anticipation of the prey's firesale affect the market depth the prey has access to, which in turn facilitates predatory trading. The two-way relationship between predatory trading and liquidity generates multiple equilibria. I show that the negative feedback loop which leads to predatory trading materializes in equilibrium when smart investors are sufficiently risk-averse (or equivalently, if the asset is sufficiently risky).

[3]That is, long-term value investors in these papers have exogenous downward-sloping demand curves.

[4]Note that there are theories of *front-running* with rational market participants (e.g. Pritsker, 2009), whereby strategic traders exploit their advanced knowledge of a rival trader's future liquidation. In this paper, strategic traders engage in *predatory trading*, i.e. they *induce* the need for another trader to liquidate his positions.

The literature on limits to arbitrage relates traders' market liquidity to their aggregate funding liquidity (Gromb and Vayanos, 2002, Brunnermeier and Pedersen, 2009, Vayanos and Wang, 2012). The present model generates a novel link between market and funding liquidity at the *trader's level*. Specifically, in times of high risk-aversion, a trader's price impact becomes an increasing function of her (perceived) funding liquidity, while in times of low risk-aversion, a trader's price impact is independent of her funding liquidity. Another key driver of the equilibrium is the distribution of initial asset ownership. When smart investors start with a small position in the risky asset, an increase in their position increases the probability of predation, and decreases it otherwise.

The model has three periods, with a risky asset and a risk-free asset. There are three types of market participants: a finite number of predators (e.g. hedge funds, dealers) and one prey (e.g. another hedge fund), the rest of the market being made of a continuum of smart competitive investors. The prey faces a financial constraint: She must liquidate her entire portfolio if her marked-to-market wealth falls below some threshold, e.g. because this triggers margin calls or redemptions. The prey is initially long the asset, so that her financial constraint is likely to bind if the asset price falls below a certain threshold. Finally, I assume that the prey cannot hold more than a certain quantity of the risky asset, i.e. her ability to lever up is limited.[5]

Smart investors are risk-averse and seek to offload a long position in the risky asset in the market, i.e. they demand liquidity. For brevity, I will therefore refer to them as *hedgers*.[6] The predators and the prey are risk-neutral. Hence, in the absence of financial constraints, they would provide hedgers with liquidity by buying the asset. However, being finite in number, they have market power and thus ration liquidity by buying only limited quantities over time. As a result, the asset trades at a discount relative to its fundamental value, i.e. it is imperfectly liquid.

Now consider the effect of the prey's financial constraint. The predators may be tempted to buy less or even short the risky asset in order to ensure that its price is low enough and force the prey to liquidate. Such a strategy involves an opportunity cost: since the asset trades at a discount, the predators would prefer to buy the risky asset by spreading trades over time. However, there is also a benefit from predatory trading. Indeed, eliminating

[5] Both the limited borrowing capacity and the marked-to-market wealth constraint may stem from agency frictions arising in the process of delegation of funds by outside investors (Shleifer and Vishny, 1997).

[6] Hedgers may stand for market-makers trying to reduce their inventory, or insurance companies seeking to sell assets following or in anticipation of downgrades or other regulatory constraints.

the prey reduces the competition in the provision of liquidity, allowing predators to capture larger rents. Further, the prey's liquidation itself increases the demand for liquidity, which benefits the remaining liquidity providers.

I first study the prey's ability to resist predatory trading by buying the asset in a bid to support its price. This ability may be limited, first, by her leverage constraint, and second - and more interestingly - by the hedgers' anticipations about predatory trading. There are two effects. First, when hedgers expect the prey to liquidate, price impact becomes trader-specific (even though all information is symmetric). If the prey buys the asset to support its price, her trades move the price *less* than opposite orders by predators. Indeed, hedgers anticipate that for each share they sell to the prey, with some probability, that share will have to be liquidated in a firesale, reducing future liquidity. Hence selling a share to the prey provides them with only partial, temporary insurance. This reduces the gains from trading with the prey. In this sense, the reactions of rational hedgers to the possibility of predatory trading can be "destabilizing": the mere anticipation of the prey's distress reduces her ability to resist predatory trading.

The second effect is akin to a financial market run: the hedgers are more reluctant to holding the risky asset when they believe that the prey will be distressed. As a result, they are ready to sell their endowment at a lower price. This selling pressure can thus turn into a financial market run, as the hedgers attempt to reduce their asset holdings ahead of the prey's firesale.[7] If the hedgers are sufficiently risk-averse, their run may even be such that predators need not sell the asset: it may be enough for them to reduce the quantity of the asset they buy, i.e. "hoard" liquidity, and let the hedgers' trading push the prey into distress. This implies that short-selling bans may be ineffective to prevent predatory trading, and that there is no direct link between selling an asset and predatory trading.

The hedgers' risk appetite plays a key role in both effects. Their risk appetite depends on the size of their initial position, and the slope of their demand curve (i.e. the product of their risk-aversion and the asset volatility). The change in price impact and the run effect depend primarily on hedgers' risk-aversion. If the hedgers hold no initial positions in the asset, the effects are still present, and is stronger with long positions.

The size of the hedgers' initial position has a non-monotonic effect on the *likelihood* of

[7]The difference between the traditional models of market run (e.g. Bernardo and Welch (2004)) and this one is that the probability of the liquidity shock is endogenous. The liquidity shock (the prey's firesale) depends on the first-period price, which is determined in equilibrium.

predatory trading: the likelihood first increases and then decreases with hedgers' initial position. This results from two conflicting effects. On the one hand, the hedgers' behaviour can decrease the cost (to the predators) of predatory trading, because the hedgers' run is stronger. This is especially true if they start with a long position in the risky asset. On the other hand, the hedgers' initial position also affects predators' outside option, which consists in providing rather than withdrawing liquidity: if hedgers generate significant selling pressure (if they have a large enough initial position), liquidity provision is very profitable. As a result, an increase in hedgers' selling pressure (via an increase in their initial position in the risky asset) does not necessarily generate more predatory trading.

The analysis has implications for regulation and risk-management. The model predicts that a destabilizing feedback loop can occur when hedgers are sufficiently risk-averse. This prediction is in line with anecdotal evidence that predatory trading occurs during flights-to-liquidity episodes (e.g. LTCM in 1998, predatory activity among hedge funds in 2008). The analysis shows, more precisely, that flight-to-liquidity and predatory trading phenomena feed each others when hedgers are sufficiently risk-averse. If hedgers' risk-aversion in utility proxies for risk-aversion stemming from various constraints limiting the market's risk-bearing capacity, the model suggests that to avoid predatory trading, one should attempt to relax these constraints or provide additional risk-bearing capacity. Since hedgers' risk-aversion translates into high permanent price impact, and assuming that it is possible to classify assets by their coefficient of permanent price impact, another interpretation of the results is that financially-constrained strategic traders are more exposed to predatory trading risk when they hold assets with high permanent price impact.

Finally, the model has also implications for the relation between turnover, liquidity and welfare. The analysis shows that proxying for liquidity by turnover or price impact can be misleading. In a special case of the model where it is socially optimal not to trade because initial endowments are Pareto-efficient, I show that the mere presence of the prey's financial constraint can induce (predatory) trading and thus abnormally high turnover. Further, although liquidity worsens - the asset trades at a larger discount -, the prey's price impact decreases. Hence in the presence of large investors, traditional measures of market depth can be misleading to assess liquidity and welfare.

The literature on predatory trading relies either on exogenous liquidity (Brunnemeier and Pedersen (2005), Attari, Mello and Ruckes (2005), Carlin et al. (2007), Parida and Venter (2009), Laó (2010), Brunnermeier and Oehmke (2013)) or exogenous distress - and thus considers front-running (Pritsker (2009))..[8] The main contribution of this paper is to combine endogenous liquidity - via the assumption that all market participants are rational - and endogenous distress - through the assumption that the prey's liquidation depends on her marked-to-market wealth. My analysis shows that rational hedgers' optimal behaviour can make predatory trading more likely. My model is close to Pritsker's, who also considers rational market participants, but in a setting with *exogenous* distress, i.e. in which the prey is forced to liquidate at a given time, independently of her marked-to-market wealth. Considering endogenous distress allows me to link the hedgers' optimal behaviour to the probability of predatory trading. It also generates the novel state-dependent link between market liquidity and *a trader*'s funding liquidity.

Endogenous distress is also the main difference between this paper and Carlin et al. (2007) and explains why our findings differ. I find that predatory trading is likely to occur when the slope of hedgers' demand curve is steep, while Carlin et al.'s model predicts the opposite. In my setting, high price impact allows predators to move prices to induce the prey's distress. In Carlin et al. (2007), a high price impact allows the prey to retaliate against predators in a repeated interaction.

Modeling all market participants as rational also allows me to connect the literature on predatory trading to that on runs in financial markets and more generally destablizing speculation. The economic force triggering what I call run here is not a sequentiality issue as in Bernardo and Welch (2004), but the prospect of the prey's firesale (i.e. a supply shock) and of the predators' increased market power. A feature common to our models is the market's limited risk-bearing capacity. While Bernardo and Welch assume that hedgers are myopic, in my setting all market participants are rational and forward-looking. DeLong et al. (1990)'s model relies on the presence of positive feedback traders. In my model, the positive feedback stems from the fact that low marked-to-market wealth is followed by the prey's liquidation.

The paper proceeds as follows. Section 2 presents the model. Section 3 studies the special case where the hedgers have no endowment in the risky asset. Section 4 studies the case with positive endowments. Section 5 concludes. The appendix contains the proofs.

[8]Note that some of these papers include front-running under the umbrella of predatory trading.

2 Model

The model has three periods: $t = 0, 1, 2$, and a risky asset, in finite supply $S \geq 0$. It pays off a dividend $\tilde{D}_2$ at $t = 2$, with $\tilde{D}_2 = D + \tilde{\epsilon_1} + \tilde{\epsilon_2}$, $D > 0$. The innovations ϵ_1 and ϵ_2, revealed at $t = 1$ and $t = 2$ respectively, are independent and identically distributed normal variables with mean 0 and variance σ^2. I denote p_t the price of the risky asset. There is a risk-free asset in perfectly elastic supply with return r_f normalised to 0.

There are $n+1$ market participants, divided in two classes: *hedgers* and *strategic traders*. The hedgers are treated as a representative competitive trader (denoted by superscript 0) with exponential utility over final consumption. Their coefficient of absolute risk-aversion is α. The hedgers start with an endowment $X^0_{-1} \geq 0$ in the risky asset.[9]. Since they have CARA preferences, their initial wealth is irrelevant for the problem, hence I assume without loss of generality that they start with cash $B^0_{-1} = 0$.

The hedgers trade with $n \geq 2$ risk-neutral strategic traders, who start with endowments X^i_{-1}, $i = 1, ..., n$, in the risky asset and B^i_{-1} in cash. For trader $i = 0, 1, ..., n$, x^i_t denotes the time-t trade in the risky asset, while X^i_t denotes the end-of-time t position in the risky asset.[10] Strategic traders and the hedgers face the same dynamic budget constraint:

$$\forall i = 0, 1, ..., n,\ W^i_2 = C^i_2 = B^i_{-1} - x^i_0 p_0 - x^i_1 p_1 + X^i_1 D_2 \tag{2.1}$$

Strategic traders account for the impact of their trades on the price. At time 0 and 1, the hedgers set their demand for the risky asset as a function of its price, and strategic traders compete in quantities (à la Cournot) for the risky asset, taking this demand as given. Strategic traders can be seen as sophisticated investors such as prop trading desks, dealers or hedge funds, who have a superior understanding of the trading environment and the “order-flow”, and therefore internalize the impact of their own trades on the price.[11] For

[9]Hedgers may stand for a competitive market-making sector. Their endowment, in this case, represent market-makers’ aggregate inventory, which can result from a temporary order imbalance, in the spirt of Grossman and Miller (1988). Hedgers may also stand for the demand of two groups of local traders subject to endowment shocks in segmented markets, as in Gromb and Vayanos (2002).

[10]Throughout, capital letters denote positions, while small letters denote trades.

[11]For instance, investment banks often have a good understanding of the order-flow. Similarly, Perold reports that LTCM “believed that most of its trading opportunities arose as a result of dislocations in the financial markets caused by institutional demands”. The hedge fund “would build models to find mispricings created by such demands, but would also identify the reason for the mispricing before initiating a trade” (Perod (1999)).

simplicity, strategic traders' identities are observable, i.e. trading is not anonymous.[12]

The group of the strategic traders consists of one *prey* (trader 1, "she") and $n-1$ *predators*. The prey faces financial constraints, while predators do not. In particular, the prey is distressed and must liquidate her position in the risky asset when her marked-to-market wealth is lower than a threshold $\underline{V}$:

Assumption 1 *If* $B_0^1 + X_0^1 p_0 \leq \underline{V}$, *then* $X_1^1 = 0$.

The prey's liquidation consecutive to a low wealth may follow from large capital outflows as a response to a poor performance. A number of financial constraints are based on prices, e.g. VaR constraints, stop-loss thresholds or high-water marks. The relation between past performance and fund flows has been documented for both equity and debt financing.[13] Agency concerns resulting from the delegation of funds from investors to strategic traders can rationalize this behaviour: Bolton and Scharfstein (1990) show that a termination threat can arise as a disciplining device in an optimal contract, even if it exposes the agent to predation risk.

In addition to the marked-to-market wealth constraint, the prey faces a leverage constraint. Her time-0 position in the risky asset, X_0^1, is bounded above by $\bar{X}$.[14]

Assumption 2 $X_0^1 \leq \bar{X}$

For simplicity, I assume that predators are cash-rich or able to secure better funding conditions and do not face any financial constraints.[15]

Given that all market participants are informed about the prey's constraints, they take into account the possibility of her being distressed in their maximization problems. The hedgers choose trades x_0^0 and x_1^0 to maximize their utility subject to their dynamic budget

[12]See Foucault et al. (2003) and references therein for a description of non-anonymous trading environments . I discuss further the role of this assumption in the model in Section 3.3.2.

[13]For instance, open-end mutual funds experiencing large outflows after a string of poor returns exert significant price pressure in equity markets (Shleifer and Vishny (1997), Coval and Stafford (2007)). The repo market is also prone to runs (see, e.g. Gorton and Metrick (2010)).

[14]$\bar{X}$ may depend on the prey's initial cash, the first period price, and be correlated with the severity of the wealth constraint.

[15]Strategic traders such as hedge funds may have some leeway in chosing their capital structure. For instance, some hedge funds are able to impose better lock-up periods or gates to their investors than their rivalsand is optimal differentiation in strategic traders' capital structure can arise in equilibrium in an optimal contract setting (Hombert and Thesmar (2009)).

constraint, while taking prices and the prey's constraints as given. Their problem is given by:

$$\max_{x_0^0, x_1^0} - \mathbb{E}_0 \exp\left[-\alpha C_2^0\right]$$
$$s.t.\ C_2^0 = B_{-1}^0 - x_0^0 p_0 - x_1^0 p_1 + X_1^0 D_2$$
$$B_0^1 + X_0^1 p_0 \leq \underline{V} \Rightarrow X_1^1 = 0$$
$$X_0^1 \leq \bar{X}$$

Strategic traders maximize their expected wealth by choosing trades x_t^i ($t = 0, 1$ and $i = 1, ..., n$), subject to their dynamic budget constraint, the price schedule which results from the hedgers' demand and market-clearing, and the prey's financial constraints. The optimization problem of a strategic trader is given by

$$\forall i = 1, ..., n, \ \max_{x_0^i, x_1^i} \mathbb{E}_0\left[W_2^i\right]$$
$$s.t.\ C_2^i = B_{-1}^i - x_0^i p_0 - x_1^i p_1 + X_1^i D_2$$
$$\text{hedgers' demand at t=0,1}$$
$$\text{market-clearing at t=0,1}$$
$$B_0^1 + X_0^1 p_0 \leq \underline{V} \Rightarrow X_1^1 = 0$$
$$X_0^1 \leq \bar{X}$$

Since each strategic trader has price impact and is informed about the prey's financial constraints, these constraints enter not only the prey's optimisation problem, but also that of her rival strategic traders.

Strategic traders have a higher apetite for risk than hedgers. Hence, absent financial constraints, trading is motivated by the hedgers being (strictly) long the risky asset. In that case, the hedgers would offload some of the risk of this position onto the risk-neutral strategic traders. To isolate the effect of the financial constraint in the model, and show how it leads to predatory trading, I start with a special case, in which the hedgers do not initially hold the risky asset.

3 Predatory trading vs no trading

In this section, I solve the model in the case where the hedgers have no initial position in the risky asset (i.e. $X^0_{-1} = 0$), which implies that the strategic traders initially hold all the asset supply. With no risks to hedge, there should be no trading. However, the presence of the financial constraint may generate predatory trading, in particular if the hedgers have a low risk-bearing capacity. In the predatory trading equilibrium, the traders' financial strength (or at least the hedgers' perception of it) affects their price impact. In particular, I show that the prey's price impact decreases, while the predators' increases, which reduces the probability of survival of the prey.

3.1 Liquidity rationing during firesales

Since she is initially long the asset, the prey becomes distressed when the price of the asset at time 0 is low. In particular, by rearranging the terms in the marked-to-market wealth constraint, one can see that the prey is in distress when the price falls below $\bar{p}_0$, where $\bar{p}_0$, the prey's distress threshold, is given by

$$\bar{p}_0 \equiv \frac{\underline{V} - B^1_{-1}}{X^1_{-1}} \tag{3.1}$$

Note that the higher the distress threshold is, the more exposed the prey is to a forced liquidation. The threshold is increasing in $\underline{V}$, which measures the severity of the constraint, and decreasing in the amount of cash the prey initially holds, B^1_{-1}. I assume that parameters are such that $0 < \bar{p}_0 < D$, i.e.

Assumption 3 $0 < X^1_{-1} D < \underline{V} - B^1_{-1}$

This assumption implies that the prey remains solvent if the asset trades at its expected value. At time 1, all market participants are aware of whether the prey is in distress or not. The following lemma summarizes the equilibrium at time 1, depending on whether the prey is distressed or not.

Lemma 1 *When the prey is solvent, there is a unique symmetric equilibrium at time* 1*, given by:*

$$\forall i = 1, ..., n,\ x^i_1 = \frac{-\sum_{j=1}^n x^j_0}{n+1} \tag{3.2}$$

When the prey is distressed, the unique equilibrium at time 1 *is given by:*

$$\begin{aligned} x_1^1 &= -X_0^1 \\ \forall i = 2, ..., n, \ x_1^i &= \frac{\left(X_{-1}^1 + x_0^1\right) - \sum_{j=1}^n x_0^j}{n} \end{aligned} \tag{3.3}$$

Equation (3.2) shows that when the prey is solvent, strategic traders trade in the opposite direction to the time 0 aggregate order flow, $\sum_{j=1}^n x_0^j$. Note that because of imperfect competition, the total order nx_1 does not completely offset the time-0 aggregate order-flow: $|\sum_{j=1}^n x_1^j| \leq |-\sum_{j=1}^n x_0^j|$. If the prey is distressed, she no longer behaves strategically and liquidates her position by submitting an order $x_1^1 = -X_0^1$ at the prevailing market price. In other words, the prey behaves as a liquidity trader. Equation (3.3) shows that the predators take the opposite side of her trade and of the previous aggregate order flow, $\sum_{j=1}^n x_0^j$. They do so, however, only to a certain extent. Indeed, the predators gain market power and can thus limit further the quantity they trade. This can be seen by comparing equations (3.2) and (3.3): for a given supply, strategic traders' aggregate order at time 1 is a fraction $\frac{n}{n+1}$ of the supply in the no-distress case and $\frac{n-1}{n}$ of the supply in the distress case, with for all $n \geq 2$, $\frac{n}{n+1} > \frac{n-1}{n}$. I denote this effect the rationing of liquidity provision. It implies that, during a firesale, the predators do not completely offset the selling/ buying pressure of the distressed prey. Hence, in equilibrium, the hedgers will have to absorb some of the prey's asset firesale. Because there is still uncertainty at time 1 about the fundamental value of the asset, the hedgers are unwilling to hold large quantities. Therefore, at time 0, the hedgers take into account the possibility of the prey's distress when setting their demand.

3.2 Run and asymmetric (trader-specific) price impact

Since hedgers understand that predators will ration liquidity further during firesales, their demand changes depending on whether they expect a firesale or not at time 1. This affects the properties of the price schedule (i.e. the inverted demand schedule combined with market-clearing) faced by the predators and the prey at time 0.

Lemma 2 *Let p_0^{nd} and p_0^d denote the price schedule when hedgers expect no-distress and distress, respectively. The price schedule depends on the hedgers' beliefs about future distress* *as follows:*

$$p_0^{nd} = D + \beta \frac{n+2}{n+1} \sum_{i=1}^{n} x_0^i \tag{3.4}$$

$$p_0^{d} = D + \beta \sum_{j=1}^{n} x_0^j + \beta \frac{1}{n} \left(\sum_{j=1}^{n} x_0^j - X_0^1 \right) \tag{3.5}$$

Strategic traders' identities are public information, hence, using the dynamics of asset holdings, $X_0^1 = X_{-1}^1 + x_0^1$, equation (3.5) can be rewritten as:

$$p_0^d = D - \beta \frac{1}{n} X_{-1}^1 + \beta \frac{n+1}{n} \sum_{j=2}^{n} x_0^i + \beta x_0^1 \tag{3.6}$$

Comparing equations (3.4) and (3.6) shows that when the hedgers believe that the prey will be distressed, price impact becomes trader-specific. In particular, the prey's trades now move the price less than predators', while all traders have the same price impact when the hedgers expect no distress.

The intuition for this result is that the price impact coefficients reflect the differential marginal gains from trading across different types of strategic traders. If the hedgers think that the prey will have to liquidate and anticipate that they will have to hold some of the prey's position in equilibrium (equation (3.3)), they believe that they will gain marginally less from, say, selling to the prey than to predators at time 0. Indeed, selling to predators has some advantage in terms of hedging: predators will keep this asset until time 2, i.e. until the asset pays off and returns to perfect liquidity. This is not the case when selling to the prey: if the hedgers are right, the asset sold at time 0 to the prey will return to the market at time 1, while the predators will ration liquidity.

Further, equation (3.5) shows that hedgers are now ready to sell the risky asset at a lower price than when they believe that the prey will stay in the market. For instance, consider the following thought experiment: suppose that all strategic traders buy $\hat{x} \geq 0$ in both cases. The overal impact of the trades is $\beta \frac{n^2+n-1}{n} \hat{x}$ in the (anticipated) distress case, and $\beta \frac{n^2+2n}{n+1} \hat{x}$ in the no-distress case. Since $\frac{n^2+2n}{n+1} > \frac{n^2+n-1}{n}$, and given that the constant is lower in the bad scenario, the same purchase translates into a lower price in the distress case than in the no-distress case. The intuition is simply that, in anticipation of the firesale, hedgers are unwilling to hold a long position in the risky asset.

Another way to gain intuition in this effect is to assume that predators and the prey do not trade at time 0, $\forall i = 1, .., n,\ x_0^i = 0$. Then if the hedgers believe that the prey will be solvent, the price is $p_0^{nd} = D$. Since all the asset supply is initially the hands of the predators and the prey, who are risk-neutral, the price must coincide with the expected value of the asset. If the hedgers anticipate the prey to be distressed, the price is $p_0^d = D - \beta \frac{X_{-1}}{n}$. That is, the hedgers, anticipate that the prey will have to liquidate her position, $X_{-1}^1 > 0$, and that because of the predators' liquidity rationing, they will have to hold some of this additional supply. Hence the price adjusts downwards at time 0 in anticipation of this supply shock. In particular, the more concentrated the market is (i.e. the smaller n), and / or the more risk-averse the hedgers are, the larger the discount the price will exhibit at time 0. More concentration means that a tighter rationing of liquidity in the future, which will force the hedgers to absorb more of the supply. Of course, their valuation for holding the additional supply of risky asset decreases with their risk-aversion.I summarize these results as follows:

Lemma 3 *When the hedgers expect the prey to be in distress at $t = 1$,*

- *they are ready to sell at a lower price than when they expect no distress [run] :*

$$p_0^{nd}(\hat{x}) > p_0^d(\hat{x}),\ x_0^1 = x_0^j = \hat{x} \geq 0,\ j = 2, ..., n$$

- *the prey has less price impact than predators [asymmetric price impact] :*

$$\forall j = 2, ..., n,\ \frac{\partial p_0^{nd} / \partial x_0^j}{\partial p_0^{nd} / \partial x_0^1} < \frac{\partial p_0^d / \partial x_0^j}{\partial p_0^d / \partial x_0^1}$$

Note that the run effect is stronger when the prey has a large initial position in the risky asset, since all else equal, its liquidation will hurt the hedgers more in case of distress. The fact that price schedules depend on the hedgers' expectations about the prey's distress has important consequences for the equilibrium determination: the predators' ability to move the price (and the prey's ability to counter them) vary depending on the hedgers' beliefs about future distress.

3.3 Equilibria

Taking hedgers' beliefs as given, I determine conditions under which no trading and predatory trading arise in equilibrium.

3.3.1 No trading

Suppose that hedgers anticipate no trading, and thus no distress.[16] It is never in the interest of the prey, who is risk-neutral, to exit the market. The predators, however, may have an incentive to deviate from the no-trading situation to push the prey into distress. This is costly, because it requires to manipulate the price and tighten the prey's financial constraint. But a deviating predator may benefit from the increase in the asset supply resulting from the prey's firesale, and the decrease in competition among the remaining strategic traders.

Predators' trade-off. Since all predators have price impact, each of them recognizes he is pivotal for the outcome of the game. Deviating from the no-trading strategy can be profitable, however, only if this leads to the prey's distress, which require to push the price to $\bar{p}_0$. A predator thus faces a trade-off between manipulating the price and gaining from the prey's firesale. The predator's problem is:

$$\begin{array}{rcl} \forall i=2,...,n, \quad \max_{x_0^i} & & E_0\left(B_{-1}^i - x_0^i p_0 + CT_1^i\right) \\ s.t.\ p_0^{nd} & = & D + \beta \dfrac{n+2}{n+1}\displaystyle\sum_{i=1}^{n} x_0^i \\ \forall j \neq i,\ x_0^j & = & 0 \\ p_0 \leq \bar{p}_0 & \Rightarrow & x_1^1 = -X_{-1}^1 \end{array}$$

CT_1^i denotes the continuation payoff of the predator, which is contingent on the prey's distress. From equations (3.2) and (3.3), I get:

$$CT_1^i = \begin{cases} X_0^i D + \frac{\left(-\sum_{j\neq i} x_0^j - x_0^i\right)^2}{(n+1)^2} & \text{if } p_0 > \bar{p}_0 \\ X_0^i D + \frac{\left(X_{-1}^1 - \sum_{j=2,j\neq i}^{n} x_0^j - x_0^i\right)^2}{n^2} & \text{if } p_0 \leq \bar{p}_0 \end{cases}$$

Using the price schedule and the conjectured strategy for the other strategic traders, the predator's problem can be rewritten as follows:

$$\forall i=2,...,n,\ \max_{x_0^i} E_{-1}^i + \beta\left[\underbrace{-\frac{n+2}{n+1}\left(x_0^i\right)^2}_{t=0\ cost} + \underbrace{\frac{\left(-x_0^i\right)^2}{(n+1)^2} I_{p_0>\bar{p}_0}}_{profit\ if\ solvent} + \underbrace{\frac{\left(X_{-1}^1 - x_0^i\right)^2}{n^2} I_{p_0\leq\bar{p}_0}}_{profit\ if\ distressed}\right],$$

[16] From equation (3.4), if all strategic traders do not trade (i.e. submit orders $x_0^i = 0$), the asset will trade at the fundamental value - and therefore the prey will not be distressed.

with $E^i_{-1} = B^i_{-1} + X^i_{-1}D$, the expected value of the predator's endowment, and I_c a dummy variable that equals one when the condition c is satisfied. This maximization problem illustrates the predator's trade-off. If the predator chooses $x^i_0 = 0$, the price will be above the prey's distress threshold $\bar{p}_0$, and the predator's profit is thus 0[17]. If the predator chooses to push the price down to $\bar{p}_0$, he can benefit at time 1 from the decreased competition and the prey's firesale - the numerator of the profit in the distressed case is n^2 instead of $(n+1)^2$, and the numerator increases by $X^1_{-1} > 0$, the prey's initial position in the asset. However, to trigger the prey's distress, he must short the asset, and this involves a quadratic cost at time 0, $\frac{n+2}{n+1}\left(x^i_0\right)^2$.

Ruling out "self-fulfilling" distress. By inspecting the maximization problem, one can also see that the prey's distress can be "self-fulfilling". Namely, ex-ante, it is optimal to take a short position in the asset if one expects a negative supply shock in the future (i.e. the prey's firesale).[18] Since the predators' trades affect prices, the anticipation by a predator that the prey will be distressed at time 1 may indeed lead to a price below $\bar{p}_0$ and trigger the prey's distress. The self-fulfilling distress can be defined more formally as follows:

Definition 1 *Suppose that strategic traders $-i$ choose $x^{-i}_0 = 0$. The prey's distress is self-fulfilling if $p_0\left(\hat{x}^i_0\right) \leq \bar{p}_0$, where*

$$\hat{x}^i_0 = \arg\max_{x^i_0} E^i_{-1} + \beta\left[-\frac{n+2}{n+1}\left(x^i_0\right)^2 + \frac{\left(X^1_{-1} - x^i_0\right)^2}{n^2}\right]$$

To focus on predatory trading as a strategy aiming at eliminating a rival trader, I rule out self-fulfilling distress by imposing the following condition throughout:

Lemma 4 *There is no self-fulfilling distress if and only if $\beta < \bar{\beta}_{nd}$, where*

$$\bar{\beta}_{nd} = \frac{D - \bar{p}_0}{h_n X^1_{-1}}, \ \textit{with} \ \ h_n = \frac{n+2}{n^3 - 2n^2 - n + 1}$$

[17] Note that since $\forall n \geq 2$, $\frac{n+2}{n+1} > \frac{1}{(n+1)^2}$, all other strategies leading to $p_0 > \bar{p}_0$ are dominated by $x^i_0 = 0$.

[18] More specifically, if the predator "anticipates" the prey's distress, he expects an increase in the asset supply and less competition in the future. Therefore the marginal cost of buying one more unit at time 1 decreases. Hence it is optimal for the predator to buy less at time 0 (i.e. here, short the asset) and exploit the negative price pressure exerted by the prey's firesale at time 1.

On this parameter interval, inducing distress requires a predator to trade

$$x_0^i = \frac{n+1}{n+2}\frac{\bar{p}_0 - D}{\beta} < 0 \tag{3.7}$$

To rule out self-fulfilling distress, one must focus on situations in which the hedgers' demand curve has a flat enough slope, i.e. if $\beta < \bar{\beta}_{nd}$. Intuitively, in this case, the price is not responsive enough to trades, such that a short position taken by a trader anticipating distress does not automatically lead to the prey's firesale. The predator's order, given by equation (3.7) is just enough to push the price to $\bar{p}_0$.

Proposition 1 *There exists a no-trading equilibrium in which the prey remains solvent if and only if* $\beta < \underline{\beta}_{nd}$, *with* $0 < \underline{\beta}_{nd} < \bar{\beta}_{nd}$. *Equilibrium prices are:*

$$p_0 = D \tag{3.8}$$

$$p_1 = D + \epsilon_1 \tag{3.9}$$

This result shows that the no-trading equilibrium holds in the presence of financial constraints only if the slope of the hedgers' demand curve is flat enough. Intuitively, if the slope is steep, a predator can easily move the price against the prey - see equation (3.4) - and this reduces the cost of predation - equation (3.7). Further, a steep slope means that hedgers are reluctant to bear risk (or equivalently that the asset is very risky), implying that the firesale exerts a strong negative pressure on the price at time 1.

3.3.2 Predatory trading

I now assume that the hedgers believe at time 0 that the prey will be distressed in the future. As shown above, the price schedule in this case is:

$$p_0^d = D + \beta \sum_{j=1}^{n} x_0^j + \beta \frac{1}{n} \left(\sum_{j=1}^{n} x_0^j - X_0^1 \right) \tag{3.10}$$

I conjecture that there exists an equilibrium with predatory trading in which the prey's and the predators' strategies are given by:

$$x_0^1 = \bar{X} - X_{-1}^1 \tag{3.11}$$

$$\forall i = 2, ..., n,\ x_0^i = \frac{1}{n-1}\left[X_{-1}^1 + \frac{n}{n+1}\left(\frac{R}{\beta} - \bar{X}\right)\right] \text{ with } R = \bar{p}_0 - D \tag{3.12}$$

These strategies are constructed in a way that, in equilibrium, (i) it is too costly for the prey to stay in the market (i.e. keep the price above $\bar{p}_0$); (ii) in particular, the prey' leverage constraint is binding, and (iii) the predators push the price to the distress threshold $\bar{p}_0$. Further, I continue to assume that the prey's distress is not self-fulfilling. Since the price schedule is different, the condition under which one can rule out self-fulfilling distress are also slightly different:

Lemma 5 *Denote $a = \frac{\bar{X}}{X_{-1}^1}$ the prey's leverage capacity (i.e. $a \geq 1$). Predatory trading is*

- *never self-fulfilling if $a > \bar{a}_n$, where $\forall n \geq 2$, $\bar{a}_n > 1$.*
- *not self-fulfilling if and only if $\beta < \bar{\beta}_d$, if $a \leq \bar{a}_n$, where*

$$\bar{\beta}_d = \frac{D - \bar{p}_0}{\rho_{0,n-1} X_{-1}^1 - d_n \bar{X}}, \text{ with } \bar{a}_n = \frac{(n+1)^2}{n^2 - n + 2}$$

The lemma shows that the prey's distress can not stem from a self-fulfilling predatory trading strategy if her leverage capacity, a, is large enough. If the prey has enough dry powder, she does not "automatically" fall into distress, because her trades support the price sufficiently. If the prey has little dry powder, i.e. a low, the prey's distress is not self-fulfilling as long as the hedgers' demand curve is not too steep, i.e. if the price is not too responsive to trades.

The prey's problem. The predators' strategy implies that it is too costly for the prey to stay in the market: holding more that $\bar{X}$ in a bid to push the price above $\bar{p}_0$ and avoid distress is infinitely costly for the prey. As a result, the prey's problem is to maximize the proceeds of liquidating her holdings. Taking predators' strategy as given, the prey's problem is:

$$\max_{x_0^1} B_{-1}^1 - x_0^1 \left[\bar{p}_0 - \beta\left(\bar{X} - X_{-1}^1 - x_0^1\right)\right] + X_0^1 \left[D - \beta \frac{1}{n+1}\left(\bar{X} - \frac{R}{\beta}\right)\right]$$

The prey's liquidation problem involves a simple trade-off between liquidating at time 0 at $\bar{p}_0 - \beta\left(\bar{X} - X_{-1}^1 - x_0^1\right)$, or at time 1 at (on average) $D - \beta\frac{1}{n+1}\left(\bar{X} - \frac{R}{\beta}\right)$. Of course, the prey's trade moves the price. If she starts selling from time 0, she will push the price below her distress threshold $\bar{p}_0$. At time 1, however, the average price depends on the prey's position only through predators' strategy, i.e. $\bar{X}$ in this case. This is because trades impact the price permanently.[19] Since the prey exactly offsets her time 0 position, $X_0^1 = X_{-1}^1 + x_0^1$, at $t = 1$, her time 0 trade has no effect on the equilibrium price at time 1. It is optimal for the prey to be fully leveraged under the following condition:

Lemma 6 *(prey's optimal liquidation strategy) The prey's best response to predators' conjectured strategy is* $x_0^1 = \bar{X} - X_{-1}^1$ *if* $\beta < \beta_F$*, with* $\beta_F = \frac{D-\bar{p}_0}{\frac{n+2}{n}\bar{X} - \frac{n+1}{n}X_{-1}^1}$.

When $\beta \geq \beta_F$, the prey's trade is $\frac{n}{n+1}\frac{D-\bar{p}_0}{\beta} + \frac{n}{2(n+1)}\bar{X} - \frac{1}{2}X_{-1}^1$, i.e. the prey either buys a small amount (if $\frac{n}{n+1}\frac{D-\bar{p}_0}{\beta} + \frac{n}{2(n+1)}\bar{X} \geq \frac{1}{2}X_{-1}^1$) or starts liquidating her position. It is easy to see that this leads to a price below $\bar{p}_0$. This strategy, combined with the predators' conjectured strategy, cannot form an equilibrium: the predators would have an incentive to deviate and sell a bit less while keeping the price below $\bar{p}_0$, because their benefit would be unchanged.[20] Similarly, there cannot be an equilibrium in which the strategies are such that the prey holds less than $\bar{X}$, the predators more than equation (3.12), and the price is less than or equal to $\bar{p}_0$. In this case, the prey would have an incentive, and enough financial slack, to deviate and outbid predators in order to stay in the market. Hence, the only possible predatory equilibrium strategies are those given by equations (3.11)-(3.12). From Lemma 5 and 6, the relevant parameter space for these strategies is $\beta \in \left]0, \bar{\beta}_d \wedge \beta_F\right[$. I show in the appendix that in the special case where $X_{-1}^0 = 0$, $\beta_F < \bar{\beta}_d$, so that the relevant interval is $\beta \in]0, \beta_F[$.

Equilibrium. In the conjectured equilibrium strategy, the prey is fully leveraged and has no interest in holding less than $\bar{X}$ (since $\beta < \beta_F$). Hence it is enough to analyze predators' trade-off to determine the equilibrium conditions. Using the same notations as before, and

[19] This can be seen from equation (6.3) in the appendix.

[20] Hence, a more "continuous" constraint, in which the amount of selling would depend on the severity of the price drop, may lead to some early liquidation for the prey.

the results of the preliminary analysis, I get the trade-off faced by a predator:

$$\begin{aligned}
\forall i = 2, ..., n, \ \ \max_{x_0^i} E_{-1}^1 + \beta \left[x_0 \frac{n+1}{n} \left(X_{-1}^1 - \sum_{j=2, j\neq i}^{n} x_0^j - x_0^i \right) \right] & \\
+\beta \frac{\left(-\sum_{j=2,j\neq i}^{n} x_0^j - x_0^i \right)^2}{(n+1)^2} I_{p_0 > \bar{p}_0} & \\
+\beta \frac{\left(X_{-1}^1 - \sum_{j=2,j\neq i}^{n} x_0^j - x_0^i \right)^2}{n^2} I_{p_0 \leq \bar{p}_0} & \quad (3.13) \\
s.t. \ \forall j \neq i, \ x_0^j = \frac{1}{n-1} \left[X_{-1}^1 + \frac{n}{n+1} \left(\frac{R}{\beta} - \bar{X} \right) \right] &
\end{aligned}$$

The first line of the maximand shows that, at time 0, the predator faces a quadratic cost, $\beta \frac{n+1}{n} \left(x_0^i \right)^2$. The second line represents the benefit from deviating from the predatory attack. Since other predators' trades exert negative pressure on the price, $\sum_{j=2,j\neq i}^{n} x_0^j$ is different from zero. If the predator joins the attack, he will, however, benefit from the firesale and the reduced competition in liquidity provision at time 1. Thus a predator "trades-off" the negative price pressure exerted by other predators at time 0, $\sum_{j=2,j\neq i}^{n} x_0^j$, against the future price pressure exerted by the prey in the following period. If the predator decides to buy while other predators attack the prey, he will rescue the prey, and therefore loses the benefit of the firesale.[21] The equilibrium is as follows:

Proposition 2 *There exists a predatory trading equilibrium characterized by equations* (3.11)-(3.12) *iff* $\beta \in \left[\underline{\beta}_d \wedge \beta_F, \beta_F \right[$, *with* $\underline{\beta}_d > 0$.

The intuition for this result is simple. If the hedgers' demand curve is steep enough, inducing the prey's distress is not too costly, hence predators engage in predatory trading against the prey. Further, in this case, the prey's firesale is likely to exert strongly negative price pressure, since the hedgers have a limited risk-bearing capacity.

The following comparative static obtains:

Corollary 1 *The equilibrium threshold* $\underline{\beta}_d$ *is lower when the prey is more exposed to the risk of forced liquidation (high* $\underline{V}$*) or has less cash (low* B_{-1}^1*).*

[21] I show this point formally in the proof of Proposition 2. Observe also, that since each predator is pivotal, there is no possibility of free-riding on the attack of other predators, especially because the conjectured predatory trading strategies are such that the first-period price reaches exactly $\bar{p}_0$. Predatory trading requires full coordination of the predators in the model.

If the prey is more constrained, the cost of the predatory trading strategy is lower, hence the condition on β is less strict.

Since the interval $\left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$ is potentially empty, there can be a concern about the existence of this equilibrium. More generally, given that equilibria depend on the hedgers' beliefs, both types of equilibria may coexist, reducing the predictive ability of the model. To illustrate the results and address these concerns, I study a numerical example.

3.3.3 Coexistence of no-trading and predatory trading equilibria

From Proposition 1 and 2, I get:

Proposition 3 *When $X_{-1}^{0} = 0$,*

- *The no-trading equilibrium is the only equilibrium for $\beta \in \left]0, \min\left(\beta_F, \underline{\beta}_d, \underline{\beta}_{nd}\right)\right]$.*
- *It coexists with the predatory trading equilibrium on $\left]\min\left(\beta_F, \underline{\beta}_d, \underline{\beta}_{nd}\right), \min\left(\beta_F, \underline{\beta}_d, \underline{\beta}_{nd}\right)\right[$.*
- *Predatory trading is the only equilibrium on $\left[\min\left(\beta_F, \underline{\beta}_d, \underline{\beta}_{nd}\right), \beta_F\right[$.*

To understand further in which circumstances equilibria may coexist and when predatory trading is the only equilibrium, I consider:

$$\bar{\beta}_d - \beta_F = \frac{D - \bar{p}_0}{\bar{X}} f\left(n, a\right)$$

where the function f is given by equation (9.1) in the appendix. The predatory trading equilibrium is the only equilibrium on a non-empty interval if $f\left(n, a\right) > 0$. Since f is monotonically increasing in a, the function implicitly defines a cutoff $a^*\left(n\right)$ such that:

$$f\left(n, a^*\left(n\right)\right) = 0$$

Hence the predatory trading equilibrium exists if $a \leq a^*\left(n\right)$. Panel (a) of Figure 1 plots the cutoff a^* (red dotted line), and shows that the predatory trading equilibrium exists when both the number of predators and the prey's leverage capacity are small. Intuitively, if there are many predators, fierce competition during the prey's firesale will quickly erode the benefit of predatory trading - and more quickly than it decreases the cost per predator. Hence coordination on the predatory trading equilibria is more difficult to obtain. When the prey has a high leverage capacity, the cost of inducing distress is high, hence predatory trading is less likely.

The panel (a) of Figure 1 also features a second cutoff $\hat{a}^*(n)$ defined as

$$g(n, \hat{a}^*(n)) = 0, \text{where } \underline{\beta}_{nd} - \underline{\beta}_d = \tfrac{D-\bar{p}_0}{\bar{X}} g(n,a)$$

Since g is monotonically decreasing in a, the no-trading and predatory trading equilibria coexist (that is, $\underline{\beta}_{nd} > \bar{\beta}_d$) when $a \geq \hat{a}^*(n)$, i.e. in the region above the full dark blue line. Hence, it is only when the prey is very constrained in terms of leverage, and the group of predators very concentrated that predatory trading is the only equilibrium. The model therefore delivers a clear prediction in this case, in spite of the self-fulfilling nature of the equilibria.

In the region defined by $a \leq \hat{a}^*(n)$, the model produces the "net" probability of predatory trading (i.e. excluding the region where both equilibria coexist). The following comparative obtains:

Corollary 2 *Suppose that $a \leq \hat{a}^*(n)$ and denote $q(n,a) = 1 - \frac{\underline{\beta}_{nd}}{\beta_F}$. q decreases linearly in a, the prey's leverage capacity.*

It is costly to engage in predatory trading against the prey if she has a lot of dry powder. Hence the probability of predatory trading q decreases in a. To understand the effect of the number of predators, I plot q in Panel (b) of Figure 1. The graph shows that the probability decreases with n, the number of predators, and decreases faster when n is small, a non-linear effect. This is because the benefit of predatory trading decreases as $\frac{1}{n^2}$.

3.4 Changing liquidity and the cost of predatory trading

The cost of predatory trading is to push the asset price to the prey's liquidation threshold $\bar{p}_0$, while there are no other motives to trade, if only to short the asset, which has a positive expected payoff. Hence we can define the cost of predation as the distance between the predators' aggregate trade $Q = \sum_{i=2}^{n} x_0^i$ and zero. To understand how the change in price schedule affects the cost of predatory trading, it is interesting to compare the cost that prevails when the hedgers (correctly) anticipate distress, and the cost that predators would have to bear if the hedgers mistakenly believed that the prey will not liquidate. To make this comparison, I fix the prey's strategy and assume that she is fully leveraged, as it is a feature of any predatory equilibrium.[22]

Lemma 7 *Suppose $X_0^1 = \bar{X}$, and let Q^d denote the cost of trading when hedgers anticipate distress and Q^{nd} when they do not. For all parameter values, predators must short less when the hedgers anticipate distress, $Q^d \geq Q^{nd}$, with $Q^{nd} < 0$.*

This result shows that it becomes cheaper for predators to push the prey into distress when hedgers anticipate that the prey will eventually be forced to liquidate her positions. Each unit bought by the prey pushes up the price less than an opposite order by a predator. The asymmetric price impact reflects the hedgers' perceptions of the different traders' financial strength. It depends on the prey's financial condition being known by other traders. Although this effect has not been tested yet, there is some incidental evidence in Cai (2009), who finds that LTCM' price impact was on average lower in the months before receiving margin calls in September 1998 than during the crisis itself.

Another interesting implication of the change in liquidity is that the size of the prey's initial position has an ambiguous effect on predators' time 0 trade, i.e. on the cost of predatory trading:

Corollary 3 *Denote $\bar{X} = aX_{-1}^1$, with $a \geq 1$. Then from equation (3.12), the effect of a change in the prey's initial size on predators' aggregate order Q^d is:*

$$\frac{\partial Q^d}{\partial X_{-1}^1} = \underbrace{1}_{run\ effect\ >0} + \underbrace{\frac{n}{n+1}}_{diff.\ price\ impact\ \text{“multiplier”}\ <1} \underbrace{\left[-a + \frac{1}{\beta}\frac{\partial R}{\partial X_{-1}^1}\right]}_{collateral\ effect\ <0}$$

where $R = \bar{p}_0 - D$.

Corollary 3 describes the impact of a small change in the prey's position on the amount predators must trade to push her into distress. The corollary shows that holding a large position in the risky asset may either decrease or increase the cost of predatory trading. Holding a large position strengthens the run effect, because the hedgers anticipate a larger firesale in the following period, and the price has to adjust further downwards ex-ante. This

[22] The condition for this strategy to be optimal given that predators engage in predation would be different. In particular the interval on which this strategy is optimal would decrease. Denoting $\tilde{\beta}_F$ the threshold under the incorrect beliefs, I show in the proof of Lemma 7 that $\tilde{\beta}_F < \beta_F$. I also show that $\bar{\beta}^d > \tilde{\bar{\beta}}^d$, i.e. there is a larger interval under which distress is not self-fulfilling. Because equilibrium conditions change, my result is about the cost of predatory trading, and not the probability of predatory trading.

makes it easier for predators to trigger financial distress. At the same time, a larger position means that the prey is richer and that her distress threshold is lower - see equation (3.1), which makes predatory trading more costly. Interestingly, the run effect is 1, while the collateral effect is multiplied by $\frac{n}{n+1} < 1$. This is a consequence of the decrease in price impact the prey experiences in this regime. Hence the decrease in price impact reduces the benefit of holding a large position.

3.5 Implications for liquidity measures

Our analysis has interesting implications for liquidity measures and liquidity proxies. First, from the example above, it is clear that turnover cannot be used as a proxy for liquidity. In the absence of the prey's financial constraints, it is optimal not to trade since the more risk-tolerant investors (the prey and the predators) initially hold the entire asset supply. In that sense, the mere presence of the financial constraint generates "excessive" trading volume. There is a large literature on trading volume and excess trading volume. Heterogeneous information (e.g. Karpoff, 1986) or career concerns (Dasgupta and Prat, 2006) can increase trading volume, among other mechanisms. Here it is the financial constraint and the possibility of default that leads to an increase in trading volume. Interestingly, it is precisely when risk-aversion is high, that is when hedgers are the most unwilling to hold the asset that they end up with some in their hands.

As shown in Lemma 2, predators' price impact increases and the prey's decreases in the predatory trading equilibrium relative to the no-trading equilibrium. Further, the aggregate price impact decreases in the sense that if all traders submit the same order, it pushes up the price less when the hedgers expect a firesale than when they do not. In spite of this, one cannot conclude that the market is more liquid. In our context, trading volume and market depth can thus be misleading indicators of market liquidity. The only consistent measure is the deviation of the transaction price from the risk-neutral value of the asset $\mathbb{E}(D_2)$.

4 Predatory trading vs liquidity provision

I now move on to the case where the hedgers start with a long position in the risky asset, i.e. $X_{-1}^0 > 0$. Strategic traders hold the remainder of the supply, and the prey has a long initial position $X_{-1}^1 > 0$. The main effect of strictly positive endowments for the hedgers is to introduce a trading motive between strategic traders and hedgers based on risk-sharing. Thus the no-trading equilibrium is replaced by an equilibrium with imperfect liquidity provision but no distress. In addition, I show that (i) the run effect increases with the hedgers' endowment, decreasing the cost of pushing the prey into distress for predators. At the same time, an increase in the hedgers' endowment increases the benefit of providing liquidity to the hedgers. Because of these conflicting effects, an increase in the hedgers' endowment has an ambiguous impact on the probability of predatory trading. (ii) Run and predatory trading can be so mutually-reinforcing that predators may not have to sell in order to induce the prey's distress: it may be enough for them to hoard liquidity and let the hedgers' run decrease the price.

4.1 Equilibria

4.1.1 Liquidity provision

I conjecture that there exists an equilibrium in which all strategic traders buy the asset from the hedgers, thereby providing them with liquidity (that is allowing them to swap the risky, illiquid asset for the safe, liquid asset).

Proposition 4 *Suppose* $0 < \beta < \bar{\beta}_{nd}$*. On this interval, there exists a unique (symmetric) no-distress equilibrium given by*

$$\forall i = 1, ..., n, \; x_0^i = c_{0,n} X_{-1}^0 \quad (4.1)$$

$$x_1^i = c_{1,n} X_{-1}^0 \quad (4.2)$$

iff $\beta < \underline{\beta}_{nd} \wedge \bar{\beta}_{nd}$ *and* $c_{0,n} X_{-1}^0 \leq \bar{X} - X_{-1}^1$.

Equilibrium prices are:

$$p_0 = D - \beta \rho_{0,n} X_{-1}^0 > \bar{p}_0 \quad (4.3)$$

$$p_1 = D + \epsilon_1 - \beta \rho_{1,n} X_{-1}^0 \quad (4.4)$$

with, $\forall n \geq 1$, $c_{0,n} > c_{1,n}$, $\rho_{0,n} > \rho_{1,n}$, $n \left(c_{0,n} + c_{1,n} \right) < 1$.

The coefficients $c_{0,n}$*,* $c_{1,n}$*,* $\rho_{0,n}$ *and* $\rho_{1,n}$ *are given by equations* (7.2)-(7.5)*, and the thresholds* $\underline{\beta}_{nd}$ *and* $\bar{\beta}_{nd}$ *by equations* (7.16) *and* (7.7) *in the appendix.*

The equilibrium conditions on β given in Proposition 4 are similar to those of Proposition 1, except that the thresholds $\underline{\beta}_{nd}$ and $\bar{\beta}_{nd}$ are now evaluated for $X_{-1}^0 > 0$.[23] The condition

[23] I should have written $\bar{\beta}_{ND}^0$ in the zero-endowment case of the previous section. I use the same notations in this section, by a slight abuse of notation.

$c_{0,n} X_{-1}^0 \leq \bar{X} - X_{-1}^1$ ensures that the equilibrium strategy is feasible for the prey, in spite of her leverage constraint.

The equilibrium has two main features. First, strategic traders ration liquidity in the market. In total, they buy an amount $n (c_{0,n} + c_{1,n}) X_{-1}^0$, which is lower than the hedgers endowment $(n (c_{0,n} + c_{1,n}) < 1, \forall n \geq 2)$. This follows from the oligopolistic nature of the liquidity supply side of the market. Nevertheless, the liquidity rationing is not such that the prey is distressed: the equilibrium price is above $\bar{p}_0$. Second, strategic traders buy the asset slowly, i.e. they spread their trades over both periods. Since trades move prices in a permanent manner, a strategic trader lowers his average purchase price by splitting up trades. However, even with limited competition, there is some pressure to buy ahead of other strategic traders while the price is low. As a consequence, the first period trade is higher than the second period trade: $c_{0,n} > c_{1,n}$ for all $n \geq 2$, and even more so as n increase, as shown by Figure 2.

While strategic traders do not engage in predatory trading for $\beta \leq \bar{\beta}_{nd}$, the market is not perfectly liquid on this parameter interval. The risky asset trades at a discount because of imperfect competition and the ensuing rationing of liquidity provision. This discount decreases over time because of the gradual purchases of the strategic traders, and varies as follows:

Corollary 4 *The illiquidity discount in period t is $\Gamma_t = E_t (D_2) - p_t = \beta \rho_{t,n} X_{-1}^0 > 0$ $(t = 0, 1)$.*

- *At each period, the discount is larger for a higher risk-aversion coefficient α, a higher riskiness of the asset σ^2, a larger hedging need X_{-1}^0, a smaller number n of strategic traders.*

- *The discount decreases faster when n is small.*

The effect of the number of strategic traders on the speed at which the discount decreases is illustrated by Figure 2. The slow adjustment of the price is typical of a "gradual arbitrage", as in Oehmke (2010), except that the illiquidity of the market is endogenous in the present setting. The main driver of this phenomenon is imperfect competition. The perfect competition case, which obtains in the limit case $n \to \infty$, offers an interesting benchmark:

Corollary 5 *When $n \to \infty$, the strategic traders' total first period purchase converges to X_{-1}^0, the hedgers endowment. Their second period total purchases converges to 0. As a* *consequence, the illiquidity discount goes to 0, strategic traders' trading profits go to 0 and the hedgers certainty equivalent converges to the expected value of his endowment.*

Hence when perfect competition among strategic traders obtains, the market becomes perfectly liquid.

4.1.2 Predatory trading

Price schedule. Suppose that the hedgers believe that the prey will be in distress at time 1, then the price schedule is:

$$p_0^d = D - \beta\frac{n+1}{n}X_{-1}^0 - \beta\frac{1}{n}X_{-1}^1 + \beta\frac{n+1}{n}\sum_{i=2}^{n} x_0^i + \beta x_0^1 \tag{4.5}$$

Equation (4.5) shows that the constant of the price schedule decreases when hedgers have positive endowment. Hence I obtain the following comparative static:

Corollary 6 *The run effect is stronger when hedgers have a positive endowment in the risky asset.*

The intuition is that the hedgers now have a lower marginal valuation for the asset and are thus more eager to offload their risk ahead of the prey's firesale.

Equilibrium. The conjectured predatory trading equilibrium strategy is:

$$x_0^1 = \bar{X} - X_{-1}^1 \tag{4.6}$$

$$\forall i = 2, ..., n,\ x_0^i = \frac{1}{n-1}\left[X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1}\left(\frac{R}{\beta} - \bar{X}\right)\right] \tag{4.7}$$

The only difference with the no-endowment case is for predators' trade. It needs not be as low, as can see by comparing equations (4.7) and (3.12). This is because the hedgers' run is stronger, pushing the price down further.

I now study the trade-off faced by predators. The predator's maximization problem is the same as 3.13 except that the hedgers' endowment affect the cost, as well as the relative benefit of predatory trading.

$$
\begin{aligned}
\forall i = 2, ..., n, \ \max_{x_0^i} E_{-1}^i + \beta & \left[x_0^i \frac{n+1}{n} \left(X_{-1}^0 + X_{-1}^1 - \sum_{j=2, j \neq i}^{n} x_0^j - x_0^i \right) \right] \\
& + \beta \frac{\left(X_{-1}^0 - \sum_{j=2, j \neq i}^{n} x_0^j - x_0^i \right)^2}{(n+1)^2} I_{p_0 > \bar{p}_0} \\
& + \beta \frac{\left(X_{-1}^0 + X_{-1}^1 - \sum_{j=2, j \neq i}^{n} x_0^j - x_0^i \right)^2}{n^2} I_{p_0 \leq \bar{p}_0} \\
s.t. \ \forall j \neq i, \ x_0^j = & \frac{1}{n-1} \left[X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1} \left(\frac{R}{\beta} - \bar{X} \right) \right]
\end{aligned}
\tag{4.8}
$$

By comparing the maximization problems 4.8 and 3.13, one can see that the cost of predatory trading will be lower (first line). This is caused by the fact that the hedgers run more strongly when they have positive endowments. The benefit from resucing the prey (second line) is higher, and the benefit from predatory trading too. I show in the appendix that the trade-off faced by predators has a simple quadratic form. A predator joins the predatory trading attack if and only if

$$
a_d \beta^2 + b_d \beta + c_d \geq 0 \tag{4.9}
$$

where the coefficients are given by equations (8.14)-(8.16) in the appendix. I obtain the following result.

Proposition 5 *Denote* $\theta = \frac{X_{-1}^0}{X_{-1}^1}$ *and* $a = \frac{\bar{X}}{X_{-1}^1}$*, the prey's leverage capacity. There exists an equilibrium with distress given by equations* (4.7)-(4.6) *iff* $\beta \in I_P$*, where* I_P *is as follows:*

- *If* $a \geq \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$*, then* $I_P = \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$
- *If* $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$*, then* $I_P = \left[\underline{\beta}_d, \bar{\beta}_D\right[$
- *If* $\min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right) < a < \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$*, then*
  - *If* $\theta > \theta^*$*, then* $I_P = \left[\underline{\beta}_d \wedge \beta_F, \underline{\beta}_{d,2} \wedge \beta_F\right[$*,*
  - *If* $\theta \leq \theta^*$*, then* $I_P = \left[\underline{\beta}_d \wedge \bar{\beta}_d, \bar{\beta}_d\right[$*.*

with $\underline{\beta}_d$ *and* $\underline{\beta}_{d,2}$ *the positive roots of equation* (4.9)*.*

The equilibrium price is:

$$p_0 = \bar{p}_0 \tag{4.10}$$

$$p_1 = D + \epsilon_1 - \beta \frac{\bar{X}}{n+1} - \frac{|R|}{n+1} \tag{4.11}$$

Proposition 5 shows that the equilibrium is driven by three factors: the prey's leverage capacity, a, the ratio $\theta = \frac{X^0_{-1}}{X^1_{-1}}$, and the number of predators (since the coefficients m_1, m_2, κ_2 are functions of n)[24] . Intuitively, θ measures the selling pressure caused by the hedgers willingness to share risk relative to that caused by the prey's firesale. The result suggests that predatory trading can occur in equilibrium whether θ is large relative to a or not, i.e. θ plays an ambiguous role. The following comparative statics confirm this observation.

4.2 Implications

4.2.1 Hedgers' endowment and probability of predatory trading

Using the results of Proposition 5, I can calculate the probability of predation. The "gross" probability is unadjusted for the fact that the liquidity provision equilibrium can coexist with the predatory trading equilibrium. The "net" probability does take into account the possible coexistence of equilibria. I obtain the following comparative statics with respect to θ.

Corollary 7 *The gross and net probabilities of predation vary as follows.*

- *If $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, denote $\kappa = \frac{\theta+1}{a}$ and define the gross probability of predatory trading $\hat{q}$ as*

  $$\hat{q}(\kappa, n) = \frac{\bar{\beta}_d - \underline{\beta}_d}{\bar{\beta}_d}$$

  $\hat{q}$ decreases in κ, i.e. $\hat{q}$ decreases with θ on this interval.

- *If θ is small, such that $a \geq \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, the equilibrium thresholds are ordered as follows: $\underline{\beta}_{nd} < \beta_F < \bar{\beta}_d \wedge \bar{\beta}_d$. Hence the net probability of predation q is*

[24] Note that $\forall n \geq 2$, $\frac{1}{\kappa_2} \leq 1$, and $\max\left(m_2, \frac{1}{\kappa_2}\right) = m_2$. Hence, given that $a \geq 1$, in the special case $X^0_{-1} = 0$, i.e. $\theta = 0$, the equilibrium condition is $\beta \in \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$, as in Proposition 1.

given by

$$q(\theta, n, a) = 1 - \frac{\beta_{nd}}{\beta_F}$$

Then for θ small, q increases with θ.

The effect of θ on the probability of predation is non-monotonic[25]. If the hedgers' initial positions relative to the prey's are sufficiently large, then increasing θ decreases the likelihood of predatory trading. However, if θ is initial small, then increasing it may *increase* the probability of predatory trading. There are two conflicting effects at work here. First, the hedgers' initial position determines the equilibrium illiquidity discount. A high discount makes it easier to push the prey into distress. Second, a large endowment raises the opportunity cost of pushing the prey into distress. This is because predatory trading aims at decreasing the price at which strategic traders can buy the asset. However, if the price is already low because the hedgers have large positions to offload, there is a low incentive to engage in predatory trading.

4.2.2 Runs, predatory trading, and short-selling

In Corollary 6, I showed that when X_1^0 is large, hedgers run more. Interestingly, the run can be so strong that the predators may not have to short the asset to trigger the prey's distress.

Corollary 8 *(liquidity hoarding) The predators' aggregate order at $t = 0$ is $Q^d = \sum_{i=2}^n x_0^i = X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1}\left(\frac{R}{\beta} - \bar{X}\right)$.*

- *If $\theta = 0$, then $\forall \beta < \beta_F$, $Q^d < 0$, i.e. predators must short the asset to push the prey into distress.*
- *If $\theta > 0$, then if the prey has a small enough leverage capacity (a small enough), there exists $\beta^h > 0$ such that for $\beta \geq \beta^h$, $Q^d \geq 0$, i.e., it is enough for predators to hoard liquidity to push the prey into distress.*

The second part of the corollary does not state whether β^h satisfies the conditions required on β for predatory trading to occur in equilibrium. However, it is easy to calculate the various

[25] I checked numerically the "net" probability of predation, i.e. taking into account equilibrium overlap, has typically the same properties as the "gross" probability.

thresholds numerically. For instance, when $\theta = 0.3$ and $a = 1.05$, parameters are such that predatory trading is the only equilibrium for $\beta \in \left[\underline{\beta}_{nd}, \bar{\beta}_d \wedge \beta_F\right[$ as long as the number of predators is between 2 and 8. Further, for these parameters, $\beta^h < \bar{\beta}_d \wedge \beta_F$, implying that it is sufficient for predators to restrict liquidity provision, and that they do not have to short the asset. Therefore, when the hedgers are sufficiently risk-averse, they behave as predators' (involuntary) accomplices. More precisely, the possibility of predatory trading induces the hedgers to run, which in turn facilitates predatory trading. Therefore the model provides a natural link between predatory trading and financial market runs. Contrary to models of financial market runs (e.g. Bernardo and Welch (2004), the liquidity shock triggering the run, i.e. the prey's firesale, is endogenous in the model.

An interesting empirical implication of the model is that it may be misleading too look at the trade direction (i.e. buy or sell) in order to identify predators. This implication is in contrast to Brunnermeier and Pedersen's model, in which predators always sell during the predatory phase (time 0 here). Another interesting implication is that short-selling bans may not always be effective in curbing predatory trading. In particular, when the hedgers are sufficiently risk-averse ($\beta \geq \beta_h$), what pushes the prey into distress is that they quickly offload their endowment and predators restrict the quantity they buy.

4.2.3 Price effects

Predatory trading involves a price manipulation in the first period in order to push the prey into distress. Therefore the illiquidity discount is larger than in the no-distress case at time 0 when predators engage in predatory trading. The price effects of predatory trading at time 1 are as follows:

Corollary 9 *In the equilibrium with distress,*

- *The illiquidity discount at $t = 1$ is larger when the prey has a larger capacity, $\frac{\partial \Gamma_1}{\partial \bar{X}} < 0$, and when the prey has more cash or a less severe constraint $\underline{V}$, $\frac{\partial \Gamma_1}{\partial |R|} < 0$.*

- *The price rebounds on average at $t = 1$ and the average rebound is stronger when the prey is less exposed to forced liquidations (e.g. has more cash, or a looser constraint $\underline{V}$), $\frac{E_0(p_1 - p_0)}{\partial |R|} > 0$, and stronger if the prey has a smaller capacity, $\frac{E_0(p_1 - p_0)}{\partial \bar{X}} < 0$.*

If the prey has a large capacity constraint, there is a large firesale at time 1, hence a large discount and a low price rebound, on average. When the prey is not very exposed to forced liquidation, inducing distress requires to push the time 0 price to a very low level. Since price impact is permanent, the time 1 price is also lower in this case. Nevertheless, the average rebound is larger. This is because decreasing the price involves to take low or short positions at time 0, therefore predators must buy more aggressively at time 1, leading to a higher rebound on average.

5 Conclusion

I study predatory trading in a model where smart competitive investors (hedgers) understand that capital-rich strategic traders may prey upon a financially constrained competitor. I show that the hedgers' reactions to the possibility of predatory trading can make predation cheaper. This reaction manifests itself through a change in market liquidity, which allows predators to move prices more easily than the prey and increases downward pressure on the price. An important determinant of predatory trading is the hedgers' risk-bearing capacity, because it determines their ability to take the other side of predatory trades and eventually to absorb firesales without causing large market disruptions, and this determines the profitability of predatory trading.

An interesting research avenue is to study the systemic risk created by predatory trading between traders with different levels of capital. Given the mechanisms at work with one prey, one can imagine that the mere prospect of a cascade of failures could trigger a liquidity dry-up which in turn would facilitate predatory trading on multiple preys. At the same time, the possibility of becoming a prey as a result of future market disruptions may limit the willingness of traders with intermediate capital to engage in predation. Hence introducing spillovers from one prey to the other in the analysis should lead to interesting coordination problems. This is left for future research.

References

ALLEN, F., AND G. GORTON (1992): "Stock Price Manipulation, Market Microstructure and Asymmetric Information," *European Economic Review*, 36, 624–630.

ALLEN, F., L. LITOV, AND J. MEI (2006): "Large Investors, Price Manipulation, and Limits to Arbitrage," *Review of Finance*, 10, 655–693.

ATTARI, M., A. MELLO, AND M. RUCKES (2005): "Arbitraging Arbitrageurs," *Journal of Finance*, 60(5), 2471–2511.

BERNARDO, A., AND I. WELCH (2004): "Liquidity and Financial Market Runs," *Quarterly Journal of Economics*, pp. 135–158.

BOLTON, P., AND D. SCHARFSTEIN (1990): "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, 80(1), 93–106.

BRADY, N., AND ET AL. (1988): *Report of the Presidential Task Force on Market Mechanisms*. Washington: U.S. Government Printing Office.

BRUNNERMEIER, M., AND L. PEDERSEN (2005): "Predatory Trading," *Journal of Finance*, 60(4), 1825–1863.

——— (2009): "Market Liquidity and Funding Liquidity," *Review of Financial Studies*, 22(6), 2201–2238.

CAI, F. (2009): "Trader Exploitation of Order Flow Information during the LTCM Crisis," *Journal of Financial Research*, XXXII(3), 261–284.

CARLIN, B., M. SOUSA-LOBO, AND S. VISWANATHAN (2007): "Episodic Liquidity Crises: Cooperative and Predatory Trading," *Journal of Finance*, 62(5), 2235–2274.

CGFS (1999): *A Review of Financial Market Events in Autumn 1998*. Committee on the Global Financial System, Bank for International Settlements, Basle, Switzerland.

CHEN, J., S. HANSON, H. HONG, AND J. STEIN (2008): "Do Hedge Funds Exploit Mutual-Fund Distress?," Harvard Business School, Working Paper.

COVAL, J., AND E. STAFFORD (2007): "Asset Fire Sales (and Purchases) in Equity Markets," *Journal of Financial Economics*, 86(2), 479–512.

Dasgupta, A., and A. Prat (2006): "Financial Equilibrium with Career Concerns," *Theoretical Economics*, 1, 67–93.

De Long, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990): "Positive Feedback Investment Strategies and Destabilizing Rational Speculation," *Journal of Finance*, 45(2), 379–395.

Fardeau, V. (2011): "Dynamic Strategic Arbitrage," LSE Working Paper.

Foucault, T., S. Moinas, and E. Theissen (2003): "Does Anonymity Matter in Electronic Limit Order Markets?," HEC, Working Paper.

Gorton, G., and A. Metrick (2010): "Securitized Banking and the Run on Repo," Yale School of Management, Working Paper.

Gromb, D., and D. Vayanos (2002): "Equilibrium and Welfare in Markets with Financially-Constrained Arbitrageurs," *Journal of Financial Economics*, 66, 361–407.

Grossman, S., and M. Miller (1988): "Liquidity and Market Structure," *Journal of Finance*, 38, 617–633.

Hombert, J., and D. Thesmar (2009): "Limits of Limits of Arbitrage: Theory and Evidence," HEC Working Paper.

Karpoff, J. (1986): "A Theory of Trading Volume," *Journal of Finance*, 41(5), 1069–1087.

Kyle, A., and S. Viswanathan (2008): "How to Define Illegal Price Manipulation," *American Economic Review, Papers and Proceedings*, 98(2), 274–279.

La'o, J. (2010): "Predatory Trading and Credit Freeze," Massachusetts Institute of Technology, Working Paper.

Levin, C., and N. Coleman (2007): *Excessive Speculation in the Natural Gas Market*. Permanent Subcommittee on Investigations, US Senate.

Lowenstein, R. (2000): *When Genius Failed: The Rise and Fall of Long-Term Management*. Random House, New York.

Parida, S., and G. Venter (2009): "Financially Constrained Strategic Trading and Predation in Illiquid Markets," London School of Economics Working Paper.

Pérold, A. (1999): *Long-Term Capital Management, L.P.* Harvard Business Case.

Pritsker, M. (2009): "Large Investors: Implications for Equilibrium Asset Returns, Shock Absorption, and Liquidity," Finance and Economics Discussion Series, Federal Reserve Board, Washington, D.C.

Shleifer, A., and R. Vishny (1997): "The Limits of Arbitrage," *Journal of Finance*, 52, 35–55.

Weill, P. (2007): "Leaning Against the Wind," *Review of Economic Studies*, 70, 1329–1354.

Appendix

The following proofs are given in the case where the hedgers' endowment is $X^0_{-1} \geq 0$. Section 9 of this appendix contains additional derivations related to the special case where the hedgers have no endowment ($X^0_{-1} = 0$). In my derivations of the equilibrium, I use Lemma 2 in Fardeau (2011).

6 Time-1 subgame equilibrium and price schedules

Lemma 1

Proof I solve the model backwards. Given the CARA-normal framework of the model, it is convenient to work with certainty equivalents to solve the hedgers' problem.

Date 1. From the viewpoint of date 1, the first innovation ϵ_1 is known, hence $E_1\left(\tilde{D}_2\right) = D + \epsilon_1$ and the hedgers' maximisation problem is

$$CE_1 = \max_{x_1^C} B_0^C - x_1^C p_1 + X_1^C (D + \epsilon_1) - \frac{1}{2}\beta\left(X_1^C\right)^2, \text{with } \beta = \alpha\sigma^2 \tag{6.1}$$

The hedgers' demand at $t = 1$ is thus $X_1^C = \frac{D+\epsilon_1-p_1}{\beta}$. Inverting the demand curve and imposing market-clearing,

$$\forall t = 0, 1,\ S = X_t^C + \sum_{j=1}^{n} X_t^j \tag{6.2}$$

yields the price schedule faced by strategic traders:

$$p_1 = D + \epsilon_1 - \beta\left(S - \sum_{j=1}^{n} X_1^j\right)$$

Using $X_t^j = X_{t-1}^j + x_t^j$ gives:

$$p_1 = D + \epsilon_1 - \beta\left(S - \sum_{j=1}^{n} X_0^j\right) + \beta\sum_{j=1}^{n} x_1^j \tag{6.3}$$

There are two states of the world at $t = 1$, with and without distress. If there is distress, the prey must liquidate her entire portfolio, i.e. $X_1^1 = 0$, which implies $x_1^1 = -X_0^1$. Otherwise, the prey is free to choose her position.

- First case: no distress (nd). A strategic trader's value function is defined as

$$\begin{aligned}\forall i=1,...,n,\ J_1^{i,nd} &= \max_{x_1^i} E_1\left[B_0^i - x_1^i p_1 + X_1^i \tilde{D}_2\right]\\ s.t.\ p_1 &= D+\epsilon_1-\beta\left(S-\sum_{j=1}^{n} X_0^j\right)+\beta\sum_{j=1}^{n} x_1^j\end{aligned}$$

Plugging the constraint in the maximand gives:

$$\forall i=1,...n,\ J_1^{i,nd}=\max_{x_1^i} B_0^i + X_0^i\left(D+\epsilon_1\right)+x_1^i\left[S-\sum_{j=1}^{n} X_0^j-\sum_{j\neq i}^{n} x_1^j - x_1^i\right],$$

where, $\forall j=1,...,n,\ X_0^j$ has been determined in the previous period. Taking the first-order condition, solving for its zero and rearranging terms, we get:

$$\forall i=1,...,n,\ x_1^i+\sum_{j=1}^{n} x_1^j = S-\sum_{j=1}^{n} X_0^j \tag{6.4}$$

Collecting the n equations and using matrix notation gives

$$\left(I+\mathbf{1}\right).\mathbf{x_1}=\left(S-\sum_{j=1}^{n} X_0^j\right).1,$$

where $\mathbf{1}$ is a (n,n) matrix of 1's, $\mathbf{x_1}=(x_1^1,...,x_1^n)$ and 1 is a vector of 1's. The lines and columns of the matrix $A=I+\mathbf{1}$ are linearly independent. Thus the matrix is invertible with inverse A^{-1} and multiplying on both sides from the left by A^{-1} gives the unique equilibrium in the subgame:

$$\forall i=1,...,n,\ x_1^i=\frac{S-\sum_{j=1}^{n} X_0^j}{n+1} \tag{6.5}$$

Plugging this quantity into the strategic trader's value function $J_1^{i,nd}$ gives

$$J_1^{i,nd}=B_0^i+X_0^i\left(D+\epsilon_1\right)+\beta\frac{\left(S-\sum_{j=1}^{n} X_0^j\right)^2}{\left(n+1\right)^2} \tag{6.6}$$

The strategic trader's value function is the expected payoff on his date 0 positions in the riskfree and risky assets, plus the continuation payoff $\beta\frac{\left(S-\sum_{j=1}^{n} X_0^j\right)^2}{\left(n+1\right)^2}$. Using equations (6.1) and (6.5), the hedgers' certainty equivalent is:

$$CE_1^{nd} = B_0^C + X_0^C \left(D + \epsilon_1 - \beta \frac{S - \sum_{j=1}^n X_0^j}{n+1} \right) + \beta \frac{\left(S - \sum_{j=1}^n X_0^j \right)^2}{2\left(n+1\right)^2} \tag{6.7}$$

- Second case: prey is in distress (d).

In this case, $X_1^1 = 0$, hence $x_1^1 = -X_0^1$. Given that $X_1^1 = 0$, the problem of a predator is

$$\forall i = 2, ..., n, \; J_1^{i,d} = \max_{x_1^i} E_1 \left(B_0^i - x_1^i p_1 + X_1^i \tilde{D}_2 \right)$$
$$s.t. \; p_1 = D + \epsilon_1 - \beta \left(S - \sum_{i=2}^n X_1^i \right)$$

Repeating the same steps as above, I get the unique equilibrium in the subgame:

$$x_1^1 = -X_0^1 \tag{6.8}$$
$$\forall i = 2, ..., n, \; x_1^i = \frac{S - \sum_{j=2}^n X_0^j}{n} \tag{6.9}$$

Strategic trader's value function and the hedgers' certainty equivalent are given by:

$$\forall i = 2, ..., n, \; J_1^{i,d} = B_0^i + X_0^i \left(D + \epsilon_1 \right) + \beta \frac{\left(S - \sum_{j=2}^n X_0^j \right)^2}{n^2} \tag{6.10}$$
$$CE_1^d = B_0^C + X_0^C \left(D + \epsilon_1 - \beta \frac{S - \sum_{j=2}^n X_0^j}{n} \right) + \beta \frac{\left(S - \sum_{j=2}^n X_0^j \right)^2}{2n^2} \tag{6.11}$$

■

Lemma 3

Proof Date 0. I now solve for the hedgers' demand at date 0, depending on the hedgers' beliefs about the state at $t = 1$.

- First case: The hedgers believe that the prey will be solvent at $t = 0$. The hedgers' maximisation problem at $t = 0$, using $B_0^C = B_{-1}^C - x_0^C p_0$ and equation (6.7), is

$$\max_{x_0^C} E_0 - \exp -\alpha \left(-x_0^C p_0 + X_0^C \left(D + \tilde{\epsilon}_1 - \beta \frac{S - \sum_{j=1}^n X_0^j}{n+1} \right) + \beta \frac{\left(S - \sum_{j=1}^n X_0^j \right)^2}{2\left(n+1\right)^2} \right),$$

where $\tilde{\epsilon_1}$ is random. Using the projection theorem for normals, the problem simplifies to maximising the hedgers' date-0 certainty equivalent:

$$CE_0 = \max_{x_0^C} -x_0^C p_0 + X_0^C \left(D - \beta \frac{S - \sum_{j=1}^n X_0^j}{n+1} \right) + \beta \frac{\left(S - \sum_{j=1}^n X_0^j \right)^2}{2\left(n+1\right)^2} - \frac{1}{2}\frac{\beta}{2}\left(X_0^C\right)^2 \quad (6.12)$$

From the first-order condition I get the hedgers' demand function at $t = 0$:

$$X_0^C = \frac{D - \beta \frac{S - \sum_{j=1}^n X_0^j}{n+1} - p_0}{\beta}$$

Inverting the demand, imposing market-clearing (equation (6.2)):

$$p_0^{nd} = D - \beta \frac{n+2}{n+1} \left[S - \sum_{j=1}^n X_0^j \right]$$

Using the accounting identity:

$$S = X_{-1}^0 + \sum_{j=1}^n X_{-1}^j \quad (6.13)$$

gives the date-0 price functional when the hedgers anticipate no distress:

$$p_0^{nd} = D - \beta \frac{n+2}{n+1} X_{-1}^0 + \beta \frac{n+2}{n+1} \sum_{j=1}^n x_0^j \quad (6.14)$$

With $X_{-1}^0 = 0$, equation (6.14) corresponds to equation (3.4) given in the text.

- Second case: Suppose that the hedgers believe the prey will be in distress at $t = 1$. Using equation (6.11), solving for the hedgers' date 0-maximisation problem and using equation (6.13), I get:

$$p_0^d = D - \beta \frac{n+1}{n} X_{-1}^0 + \beta \sum_{j=1}^n x_0^j + \beta \frac{1}{n} \left(\sum_{j=1}^n x_0^j - X_0^1 \right)$$

Strategic traders' identities are public information, hence, using the dynamics of asset holdings, $X_0^1 = X_{-1}^1 + x_0^1$, this equation can be rewritten as:

$$p_0^d = D - \beta \frac{n+1}{n} X_{-1}^0 - \beta \frac{1}{n} X_{-1}^1 + \beta \frac{n+1}{n} \sum_{j=2}^{n} x_0^i + \beta x_0^1 \tag{6.15}$$

Setting $X_{-1}^0 = 0$ gives equation (3.5) in the text. Lemma 3 follows immediately from equations (6.14) and (6.15) and arguments given in the text. ■

7 Liquidity provision equilibrium

Lemma 4

Proof Suppose that the hedgers believe that the prey will not be distressed. Since the hegders are rational, their beliefs must be correct in equilibrium. I now determine under which condition strategic traders' actions are consistent with the hedgers' beliefs.

At date 0, a strategic trader's problem is:

$$\begin{aligned}
\forall i = 1, ..., n,\ J_0^{i,nd} &= \max_{x_0^i} E_0 \left[B_{-1}^i - x_0^i p_0 + X_0^i \left(D + \tilde{\epsilon}_1 \right) + \beta \frac{\left(S - \sum_{j=1}^n X_0^j \right)^2}{(n+1)^2} \right] \\
s.t.\ p_0^{nd} &= D - \beta \frac{n+2}{n+1} X_{-1}^0 + \beta \frac{n+2}{n+1} \sum_{j=2}^{n} x_0^j + \beta \frac{n+2}{n+1} x_0^1 \\
B_0^1 + X_0^1 p_0 &\le \underline{V} \Rightarrow X_1^1 = 0 \\
X_0^1 &\le \bar{X}
\end{aligned}$$

The second constraint corresponds to Assumption 1 (marked-to-market wealth constraint), the third constraint to Assumption 2 (leverage constraint). I first derive the equilibrium that would prevail in the absence of these two financial constraints, and then derive under which conditions this equilibrium holds in the presence of the constraints.

Ignoring the second and third constraints, plugging the first constraint into the maximand and using equation (6.13) gives

$$J_0^{i,nd} = \max_{x_0^i} E_{-1}^i + \beta \left[\frac{n+2}{n+1} x_0^i \left(X_{-1}^0 - \sum_{j \neq i}^{n} x_0^j - x_0^i \right) + \frac{\left(X_{-1}^0 - \sum_{j \neq i}^n x_0^j - x_0^i \right)^2}{(n+1)^2} \right]$$

with $E^i_{-1} = B^i_{-1} + X^i_{-1}D$. From the first-order condition, I get:

$$\forall i \in \{1, ..., n\},\ x^i_0 + \frac{n^2+3n}{(n+1)^2}\sum_{j=1}^{n} x^j_0 = \frac{n^2+3n}{(n+1)^2}X^0_{-1} \tag{7.1}$$

Solving this system of n equations with n unknowns, I get the unique equilibrium in this subgame (in absence of constraints)

$$\forall i = 1, ..., n,\ x^i_0 = \frac{n^2+3n}{n^3+4n^2+3n+2}X^0_{-1} = c_{0,n}X^0_{-1} \tag{7.2}$$

From equation (6.5), I find the date 1 equilibrium trade:

$$\forall i = 1, ..., n,\ x^i_1 = \frac{n+2}{n^3+4n^2+3n+2}X^0_{-1} = c_{1,n}X^0_{-1} \tag{7.3}$$

After some simple algebra, I obtain the equilibrium prices:

$$p_0 = D - \beta\frac{(n+2)^2}{n^3+4n^2+3n+2}X^0_{-1} = D - \beta\rho_{0,n}X^0_{-1} \tag{7.4}$$

$$p_1 = D + \epsilon_1 - \beta\frac{n+2}{n^3+4n^2+3n+2}X^0_{-1} = D + \epsilon_1 - \beta\rho_{1,n}X^0_{-1} \tag{7.5}$$

Further, using (7.2) and (7.3), I compute the payoff (skipping two lines of algebra):

$$J^{nd}_0 = E^i_{-1} + \beta\pi_{0,n}\left(X^0_{-1}\right)^2 \tag{7.6}$$

$$\text{with}\ \pi_{0,n} = \frac{(n^2+3n+1)(n+2)^2}{(n^3+4n^2+3n+2)^2}$$

Let us now consider the problem with the financial constraints. I conjecture that the equilibrium trade is given by equation (7.2). An obvious condition on parameters is that $X^1_{-1} + c_{0,n}X^0_{-1} \leq \bar{X}$.

In the presence of the financial constraints, one must check for two types of deviations. First, the prey may opt for a voluntary liquidation. The prey being risk-neutral, it is easy to show that she will never voluntarily liquidate, therefore I skip the proof.

Second, a strategic trader may turn predator and exploit the prey's constraints to trigger a forced liquidation[26]. Doing so affects strategic traders' continuation payoff, which becomes

[26]Note that since the equilibrium is unique in the absence of financial constraints, this is the only deviation

$\frac{\left(S-\sum_{j=2}^{n}X_0^j\right)^2}{n^2}$.

Let's compute the payoff from exploiting the prey's financial constraints for predator i:

$$\begin{aligned} J_0^{i,nd,dev} &= \max_{x_0^i} E_{-1}^i + \beta\left[\frac{n+2}{n+1}x_0^i\left(S-\sum_{j=1}^{n}X_0^j\right)+\frac{\left(S-\sum_{j=2}^{n}X_0^j\right)^2}{n^2}\right] \\ s.t.\ \forall j\neq i,\ x_0^j &= c_{0,n}X_{-1}^0 \\ p_0 &\leq \bar{p}_0 \end{aligned}$$

where $i\in\{2,...,n\}$. Using (6.13), this problem can be rewritten as

$$\begin{aligned} \max\nolimits_{x_0^i} \quad & \beta\frac{n+2}{n+1}x_0^i\left[X_{-1}^0-\sum_{j=1,j\neq i}^{n}x_0^j-x_0^i\right]+\beta\frac{\left[X_{-1}^0+X_{-1}^1-\sum_{j=2,j\neq i}^{n}x_0^j-x_0^i\right]^2}{n^2} \\ s.t.\ \forall j\neq i,\ x_0^j \quad &= \quad c_{0,n}X_{-1}^0 \\ p_0 \quad &\leq \quad \bar{p}_0 \end{aligned}$$

Note that in the second constraint, p_0 depends on the strategy of predator i and on the postulated strategy of other strategic traders, $\forall j\neq i,\ x_0^j=c_{0,n}X_{-1}^0$. I first determine under which condition a predatory deviation is costly, i.e. under which condition the Lagrangian of the second (price) constraint is strictly positive.

Let's first ignore the constraint $p_0\leq\bar{p}_0$ and solve for the zero of the first-order condition. I get:

$$x_0^{i,dev}=\frac{n^5+5n^4+4n^3-10n^2-11n-2}{\left(n^3+2n^2-n-1\right)\left(n^3+4n^2+3n+2\right)}X_{-1}^0-\frac{n+1}{n^3+2n^2-n-1}X_{-1}^1$$

As a consequence,

$$\frac{n+2}{n+1}\left[X_{-1}^0-\sum_{j=1}^{n}x_0^j\right]=H_1X_{-1}^0+H_2X_{-1}^1$$

with $H_1=\frac{n(n+2)\left(n^4+5n^3+8n^2+6n+3\right)}{(n+1)(n^3+2n^2-n-1)(n^3+4n^2+3n+2)}$ and $H_2=\frac{n+2}{n^3+2n^2-n-1}$. This, in turn, implies that

one must check for.

$p_0 \leq \bar{p}_0$ iff

$$\beta \geq \bar{\beta}_{nd} = \frac{|R|}{H_1 X_{-1}^0 + H_2 X_{-1}^1}, \text{with } R = \bar{p}_0 - D \tag{7.7}$$

Therefore, I will now focus on the parameter space $\beta < \bar{\beta}_{nd}$.

On this interval, pushing the prey into distress requires for a predator to set:

$$p_0^{nd} = \bar{p}_0$$

That is, predator i must choose $x_0^{i,dev}$ such that

$$D - \beta \frac{n+2}{n+1} X_{-1}^0 + \beta \frac{n+2}{n+1} \sum_{j=1, j \neq i}^{n} x_0^j + \beta \frac{n+2}{n+1} x_0^{i,dev} = \bar{p}_0$$

where $\forall j \neq i, x_0^j = c_{0,n} X_{-1}^0$. Rearranging the terms, I get:

$$x_0^{i,dev} = \frac{n+1}{n+2} \frac{R}{\beta} + \frac{2\left(n^2+3n+1\right)}{n^3+4n^2+3n+2} X_{-1}^0 \tag{7.8}$$

This achieves the proof of Lemma 4 ■

Proposition 4

Proof Building on Lemma 4, I calculate the new continuation payoff of the strategic traders.

$$X_{-1}^0 + X_{-1}^1 - \sum_{j=2}^{n} x_0^j = X_{-1}^1 + \frac{n^2+3n}{n^3+4n^2+3n+2} X_{-1}^0 - \frac{n+1}{n+2} \frac{R}{\beta} \tag{7.9}$$

Therefore, using equations (7.8) and (7.9), and developping and rearranging terms, predator

i gets the following payoff from pushing the prey into distress:

$$\begin{aligned} J_0^{i,nd,dev} &= E_{-1}^i + \beta \frac{(n+3)^2}{(n^3+4n^2+3n+2)^2} \left(X_{-1}^0\right)^2 \\ &+\beta \left[\frac{1}{n^2}\left(X_{-1}^1\right)^2 + \frac{2(n+3)}{n(n^3+4n^2+3n+2)} X_{-1}^1 X_{-1}^0\right] \\ &-R\left[\frac{2(n^4+5n^3+8n^2+6n+3)}{n(n+2)(n^3+4n^2+3n+2)} X_{-1}^0 + \frac{2(n+1)}{n^2(n+2)} X_{-1}^1\right] \\ &-\frac{(n+1)(n^3+2n^2-n-1)}{n^2(n+2)^2}\frac{R^2}{\beta} \end{aligned} \tag{7.10}$$

Hence, predator i prefers buying over preying iff $J_0^{i,nd} \geq J_0^{i,nd,dev}$. Using equations (7.6) and (7.10), it is equivalent to:

$$a_{nd}\beta^2 + b_{nd}\beta + c_{nd} \geq 0 \tag{7.11}$$

$$\text{where } a_{nd} = \lambda_1\left(X_{-1}^0\right)^2 - \lambda_2\left(X_{-1}^1\right)^2 - \lambda_3 X_{-1}^1 X_{-1}^0 \tag{7.12}$$

$$b_{nd} = R\left[\lambda_4 X_{-1}^0 + \lambda_5 X_{-1}^1\right] < 0 \tag{7.13}$$

$$c_{nd} = \lambda_6 R^2 > 0 \tag{7.14}$$

with $\lambda_1 = \frac{n^4+7n^3+16n^2+10n-5}{(n^3+4n^2+3n+2)^2}$, $\lambda_2 = \frac{1}{n^2}$, $\lambda_3 = \frac{2(n+3)}{n(n^3+4n^2+3n+2)}$, $\lambda_4 = \frac{2\left(n^4+5n^3+8n^2+6n+3\right)}{n(n+2)(n^3+4n^2+3n+2)}$, $\lambda_5 = \frac{2(n+1)}{n^2(n+2)}$, $\lambda_6 = \frac{(n+1)\left(n^3+2n^2-n-1\right)}{n^2(n+2)^2}$. Note that for all $k = 1, ..., 6,$, for all $n \geq 2$, $\lambda_k > 0$.

The discriminant of the LHS of inequality (7.11) is

$$\Delta_{nd} = R^2\left[A_1\left(X_{-1}^0\right)^2 + A_2\left(X_{-1}^1\right)^2 + A_3 X_{-1}^1 X_{-1}^0\right] \tag{7.15}$$

with $A_1 = \lambda_4^2 - 4\lambda_1\lambda_6 = \frac{4\left(3n^6+39n^5+104n^4+170n^3+125n^2+36n+4\right)}{n^2(n+2)^2(n^3+4n^2+3n+2)^2} > 0$, $A_2 = \lambda_5 + 4\lambda_6\lambda_2 > 0$, $A_3 = 2\lambda_4\lambda_5 + 4\lambda_6\lambda_3 > 0$. Hence for all $n \geq 2$, $\Delta_{nd} > 0$, which guarantees that there are always two real roots, β_1, β_2. Since the sign of b_{nd} and c_{nd} is known, the sign of equation (7.11) depends on the sign of a_{nd}.

Using $\theta = \frac{X_{-1}^0}{X_{-1}^1}$, I rewrite equation (7.12) as

$$a_{nd} = \left(X_{-1}^1\right)^2\left[\lambda_1\theta - \lambda_3\theta - \lambda_2\right]$$

The discriminant of the equation in parenthesis is $\Delta_a = \lambda_3^2 + 4\lambda_1\lambda_2 > 0$. Since $\lambda_1 > 0$ and

$-\lambda_2 < 0$, there is a positive and a negative root. The positive root is given by

$$\bar{\theta} = \frac{\lambda_3 + \sqrt{\Delta_a}}{2\lambda_1}$$

and since $\theta \geq 0$, the sign of a_{nd} is striclty negative iff $\theta \in \left[0, \bar{\theta}\right[$ and positive iff $\theta > \bar{\theta}$.

I can now determine the equilibrium:

- If $0 \leq \theta < \bar{\theta}$, the no distress equilibrium exists iff $\beta < \beta_1 \wedge \bar{\beta}_{nd}$, with $\beta_1 = -\frac{b_{nd}+\sqrt{\Delta_{nd}}}{2a_{nd}}$.
- If $\theta > \bar{\theta}$, the no distress equilibrium exists iff $\beta < \beta_1 \wedge \bar{\beta}_{nd}$ or $\beta > \beta_2 \wedge \bar{\beta}_{nd}$, with $\beta_2 = \frac{-b_{nd}+\sqrt{\Delta_{nd}}}{2a_{nd}}$.

Using equations (7.12)-(7.14), equation (7.15), and the change of variable $\theta = \frac{X^0_{-1}}{X^1_{-1}}$, the roots are given by

$$\beta_1 = \frac{|R|}{X^1_{-1}} \frac{(\lambda_4\theta + \lambda_5) - \left[A_1\theta^2 + A_3\theta + A_2\right]^{\frac{1}{2}}}{2\left(\lambda_1\theta^2 - \lambda_3\theta - \lambda_2\right)} \equiv \underline{\beta}_{nd} \tag{7.16}$$

$$\beta_2 = \frac{|R|}{X^1_{-1}} \frac{(\lambda_4\theta + \lambda_5) + \left[A_1\theta^2 + A_3\theta + A_2\right]^{\frac{1}{2}}}{2\left(\lambda_1\theta^2 - \lambda_3\theta - \lambda_2\right)} \tag{7.17}$$

I now show that in the second case ($\theta > \bar{\theta}$), the second root, β_2, does not satisfy the parameter restriction $\beta < \bar{\beta}_{nd}$, where $\bar{\beta}_{nd}$ is given by equation (7.7).

Since the denominator of β_2 is strictly positive when $\theta > \bar{\theta}$, $\beta_2 - \bar{\beta}_{nd} < 0$ is, after rearranging terms, equivalent to:

$$\left(\lambda_4 H_1 - 2\lambda_1\right)\theta^2 + \left(\lambda_5 H_1 + \lambda_4 H_2 + 2\lambda_3\right)\theta + \left(\lambda_5 H_2 + 2\lambda_2\right) + \left(H_1\theta + H_2\right)U_\theta^{\frac{1}{2}} < 0$$

where $U_\theta = A_1\theta^2 + A_3\theta + A_2$. Since for all $n \geq 2$, $\lambda_4 H_1 - 2\lambda_1 > 0$ and since all other coefficients are also positive, this condition is never satisfied for any $\theta \geq 0$, hence for any $\theta > \bar{\theta}$. Hence $\beta_2 > \bar{\beta}_{nd}$.

As a result, the necessary and sufficient condition for the existence of the no distress equilibrium is $\beta < \underline{\beta}_{nd} \wedge \bar{\beta}_{nd}$. ■

Corollaries 4 and 5

Proof The results follow directly from calculations in the proof of Proposition 4. ■

8 Predatory trading equilibrium and comparative statics

I conjecture that the predators' equilibrium predatory trade is

$$\forall j = 2, ..., n,\ x_0^j = \frac{1}{n-1}\left[X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1}\left(\frac{R}{\beta} - \bar{X}\right)\right] \tag{8.1}$$

Using equation (6.9), this implies that their date-1 trade is

$$\forall j = 2, ..., n,\ x_1^j = \frac{1}{n+1}\left(\bar{X} - \frac{R}{\beta}\right) \tag{8.2}$$

which leads to the following price:

$$p_1 = D + \epsilon_1 - \frac{\beta}{n+1}\left(\bar{X} - \frac{R}{\beta}\right)$$

I assume that the hedgers believe that the prey will be distressed. I first determine conditions under which the prey's conjectured strategy is optimal given the predators' conjectured strategy.

Lemma 6

Proof The prey's problem. The predators' conjectured strategy implies the following first-period price (as a function of the prey's trade):

$$p_0 = \bar{p}_0 - \beta\left[\bar{X} - X_{-1}^1 - x_0^1\right] \tag{8.3}$$

Since the predators' strategy is constructed so that the prey can not outbid predators, the prey's problem given predators' trade is to maximise the proceeds of liquidation. Hence the prey's maximisation problem is:

$$\begin{aligned} \max_{x_0^1} \quad & E_0 \quad \left[B_{-1}^1 - x_0^1 p_0 - x_1^1 p_1 + X_1 D_2\right] \\ s.t.\ X_1^1 &= 0 \\ x_0^1 &\leq \bar{X} - X_{-1}^1 \\ p_0 &= \bar{p}_0 = D - \beta\left[\bar{X} - X_{-1}^1 - x_0^1\right] \\ p_1 &= D + \epsilon_1 - \frac{\beta}{n+1}\left(\bar{X} - \frac{R}{\beta}\right) \end{aligned} \tag{8.4}$$

Plugging the first and last two constraints into the maximand, this problem can be rewritten as:

$$\begin{aligned} \max_{x_0^1} \quad & B_{-1}^1 - x_0^1 \left[\bar{p}_0 - \beta \left[\bar{X} - X_{-1}^1 - x_0^1\right]\right] + X_0^1 \left[D - \beta \frac{1}{n+1} \left[\bar{X} - \frac{R}{\beta}\right]\right] \\ s.t.\ x_0^1 \quad \leq \quad & \bar{X} - X_{-1}^1 \end{aligned}$$

Writing the Lagrangian of the problem and solving for the zero of the first-order condition gives:

$$x_0^1 = \begin{cases} \frac{n}{2(n+1)} \frac{|R|}{\beta} + \frac{1}{2} \left[\frac{n}{n+1} \bar{X} - X_{-1}^1\right] & \text{if } \beta < \beta_F \\ \bar{X} & \text{otherwise,} \end{cases}$$

$$\text{where } \beta_F = \frac{|R|}{\frac{n+2}{n} \bar{X} - \frac{n+1}{n} X_{-1}^1} \tag{8.5}$$

⇒ A necessary condition for the conjectured strategy to be a Nash equilibrium is $\beta < \beta_F$. ■

Lemma 5

Proof The predators' problem. The predators' conjectured strategy (8.1) is constructed assuming that predation is costly and that predators behave symmetrically. I.e., the conjectured strategy is such that predators choose a quantity leading to $p_0^d = \bar{p}_0$, with $x_0^1 = \bar{X} - X_{-1}^1$.

A necessary condition for this conjectured strategy to be a Nash equilibrium is that the Lagrangian of the first constraint in the following problem is zero.

$$\begin{aligned} \max_{x_0^i} \quad & \beta x_0^i \left[\frac{n+1}{n} \left(S - \sum_{j=2}^{n} X_0^j\right) - X_0^1\right] + \beta \frac{\left(S - \sum_{j=2}^{n} X_0^j\right)^2}{n^2} \\ s.t.\ p_0 \quad \leq \quad & \bar{p}_0 \\ X_0^1 \quad = \quad & \bar{X} \end{aligned} \tag{8.6}$$

The problem can be rewritten as

$$\max_{x_0^i} \beta x_0^i \left[\frac{n+1}{n} \left(X_{-1}^0 + X_{-1}^1 - \sum_{j=2}^{n} x_0^j \right) - \bar{X} \right] + \beta \frac{\left(X_{-1}^0 + X_{-1}^1 - \sum_{j=2}^{n} x_0^j \right)^2}{n^2}$$
$$s.t.\ p_0 \leq \bar{p}_0$$

After writing the Lagrangian of the problem and solving for the equilibrium, I get:

$$x_0^i = \begin{cases} \frac{1}{n-1} \left[X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1} \left(\frac{R}{\beta} - \bar{X} \right) \right] & \text{if } a > \frac{\rho_{0,n-1}}{d_n} \text{ or if } \beta < \bar{\beta}_d \text{ when } a \leq \frac{\rho_{0,n-1}}{d_n} \\ \frac{n^2+n-2}{n^3+n^2-2n+2} \left(X_{-1}^0 + X_{-1}^1 \right) - \frac{n^2}{n^3+n^2-2n+2} \bar{X} & \text{otherwise,} \end{cases}$$

$$\text{with } \bar{\beta}_d = \frac{|R|}{\rho_{0,n-1} \left(X_{-1}^0 + X_{-1}^1 \right) - d_n \bar{X}} \tag{8.7}$$

where $\rho_{0,n-1} = \frac{(n+1)^2}{n^3+n^2-2n+2}$, $d_n = \frac{n^2-n+2}{n^3+n^2-2n+2}$, and $a = \frac{\bar{X}}{X_{-1}^1}$ is the prey's spare leverage capacity. Note that symmetry is imposed when the Lagrangian of the constraint is zero, while it is the unique outcome when the constraint is not binding.

⇒ A necessary condition for the conjectured strategy to be a Nash equilibrium is $\beta < \beta_d$ if $a \leq \frac{\rho_{0,n-1}}{d_n}$. ■

Propositions 2 and 5

Proof The payoff of the conjectured strategy for predators is, using equations (8.1) and (8.2):

$$J_0^{i,D} = E_{-1}^i + \beta \frac{\bar{X}^2}{(n+1)^2} - R \left[\frac{1}{n-1} \left(X_{-1}^0 + X_{-1}^1 \right) - \frac{n^2-n+2}{(n-1)(n+1)^2} \bar{X} \right] - \frac{n^2+1}{(n-1)(n+1)^2} \frac{R^2}{\beta} \tag{8.8}$$

Payoff from deviating: "rescuing" the prey. Predator i may not join the predatory attack and "rescue" the prey. All predators are pivotal, hence this rescue implies a change in the continuation payoff from $\frac{S-\sum_{j=2}^{n} X_0^j}{n^2}$ to $\frac{S-\sum_{j=1}^{n} X_0^j}{(n+1)^2}$.

The strategy of a deviating predator solves the following problem:

$$\begin{aligned}
J_0^{i,d,dev} &= \max_{x_0^i} \quad \beta x_0^i \left[\frac{n+1}{n} \left(S - \sum_{j=2} X_0^j \right) - X_0^1 \right] + \beta \frac{\left(S - \sum_{j=2}^n X_0^j \right)^2}{(n+1)^2} \\
s.t.\ \forall j \neq i,\ x_0^j &= \frac{1}{n-1} \left[X_{-1}^0 + X_{-1}^1 + \frac{n}{n+1} \left(\frac{R}{\beta} - \bar{X} \right) \right] \\
X_0^1 &= \bar{X} \\
p_0 &> \bar{p}_0
\end{aligned}$$

Using equation (6.13), and plugging the first and second constraints into the maximand, the maximisation problem boils down to

$$\begin{aligned}
J_0^{i,d,dev} &= \max_{x_0^i} \quad \beta x_0^i \left[\frac{n+1}{n(n-1)} \left(X_{-1}^0 + X_{-1}^1 \right) - \frac{n-2}{n-1} \frac{R}{\beta} - \frac{n+1}{n} x_0^i - \frac{1}{n-1} \bar{X} \right] \\
&+ \quad \frac{\beta}{(n+1)^2} \left[\frac{1}{n-1} \left(X_{-1}^0 + X_{-1}^1 \right) - \frac{n(n-2)}{n^2-1} \frac{R}{\beta} - \frac{2n-1}{n^2-1} \bar{X} - x_0^i \right]^2 \\
s.t.\ p_0 &> \bar{p}_0
\end{aligned}$$

Writing the Lagrangian and solving for the first-order condition (ignoring the price constraint for now), I get the strategy of a deviating ("rescuing") predator:

$$\begin{aligned}
x_0^{i,dev} &= \frac{n^3+3n^2+n+1}{2(n-1)(n^3+3n^2+2n+1)} \left(X_{-1}^0 + X_{-1}^1 \right) \\
&- \frac{n(n^3+3n^2-n+3)}{2(n^2-1)(n^3+3n^2+2n+1)} \bar{X} - \frac{n(n-2)(n^3+3n^2+n+1)}{2(n^2-1)(n^3+3n^2+2n+1)} \frac{R}{\beta}
\end{aligned} \tag{8.9}$$

It is easy albeit algebraically tedious to check that $\beta < \bar{\beta}_d$ implies that $p_0 > \bar{p}_0$, so that the Lagrangian of the price constraint is always zero.

To compute the payoff of the rescue for predator i, it is convenient to calculate the following quantities:

$$\frac{n+1}{n} \left(X_{-1}^0 + X_{-1}^1 - \sum_{j=2}^n x_0^j \right) - \bar{X} = z_1 \left(X_{-1}^0 + X_{-1}^1 \right) - z_2 \bar{X} - z_3 \frac{R}{\beta} \tag{8.10}$$

where $z_1 = \frac{(n+1)\left(n^3+3n^2+3n+1\right)}{2n(n-1)(n^3+3n^2+2n+1)}$, $z_2 = \frac{n^3+3n^2+5n-1}{2(n-1)(n^3+3n^2+2n+1)}$, $z_3 = \frac{(n-2)\left(n^3+3n^2+3n+1\right)}{2(n-1)(n^3+3n^2+2n+1)}$. and

$$\frac{X_{-1}^0 - \sum_{j=1} x_0^j}{n+1} = z_1' \left(X_{-1}^0 + X_{-1}^1\right) - z_2' \bar{X} - z_3' \frac{R}{\beta} \tag{8.11}$$

with $z_1' = \frac{n^3+3n^2+3n+1}{2(n^2-1)(n^3+3n^2+2n+1)}$, $z_2' = \frac{3n^4+7n^3+3n^2-3n-2}{2(n-1)(n+1)^2(n^3+3n^2+2n+1)}$, $z_3' = \frac{n(n-2)\left(n^3+3n^2+3n+1\right)}{2(n-1)(n+1)^2(n^3+3n^2+2n+1)}$.

From equations (8.9)-(8.11), skipping some algebra, the payoff of rescuing the prey is:

$$\begin{aligned} J_0^{i,d,dev} = \beta \left[w_1 \bar{X}^2 + w_2 \left(X_{-1}^0 + X_{-1}^1\right) - w_3 \left(X_{-1}^0 + X_{-1}^1\right) \bar{X}\right] \\ -R \left[w_4 \left(X_{-1}^0 + X_{-1}^1\right) - w_5 \bar{X}\right] + w_6 \frac{R^2}{\beta} \end{aligned} \tag{8.12}$$

with $w_1 = \frac{n^{10}+9n^9+43n^8+114n^7+155n^6+98n^5+41n^4+50n^3+28n^2-15n+4}{4(n-1)^2(n+1)^4(n^3+3n^2+2n+1)^2}$, $w_2 = \frac{(n+1)^4}{4n(n1)^2(n^3+3n^2+2n+1)}$, $w_3 = \frac{n^6+9n^5+23n^4+24n^3+7n^2+n-1}{2(n-1)^2(n^3+3n^2+2n+1)^2}$, $w_4 = \frac{(n-2)(n+1)^3}{2(n-1)^2(n^3+3n^2+2n+1)}$, $w_5 = \frac{n^2(n-2)\left(n^5+9n^4+23n^3+25n^2+10n+4\right)}{2(n+1)(n-1)^2(n^3+3n^2+2n+1)^2}$, $w_6 = \frac{n(n-2)^2(n+1)^2\left(n^4+4n^3+4n^2+3n+1\right)}{4(n-1)^2(n^3+3n^2+2n+1)^2}$.

The conjectured predatory trades form a Nash equilibrium iff $\forall i = 2, ..., n$, $J_0^{i,d} \geq J_0^{i,d,dev}$. From equations (8.8) and (8.12), this is equivalent to

$$a_d \beta^2 + b_d \beta + c_d \geq 0 \tag{8.13}$$

$$\text{with } a_d = e_1 \bar{X}^2 - e_2 \left(X_{-1}^0 + X_{-1}^1\right)^2 + e_3 \bar{X} \left(X_{-1}^0 + X_{-1}^1\right) \tag{8.14}$$

$$b_d = -R \left[e_4 \left(X_{-1}^0 + X_{-1}^1\right) - e_5 \bar{X}\right] \tag{8.15}$$

$$c_d = -e_6 R^2 \tag{8.16}$$

and $e_1 = \frac{1}{(n+1)^2} - w_1$, $e_2 = w_2$, $e_3 = w_3$, $e_4 = \frac{1}{n-1} - w_4$, $e_5 = \frac{n^2-n+2}{(n-1)(n+1)^2} - w_5$, $e_6 = \frac{n^2+1}{(n-1)(n+1)^2} + w_6$

$$e_4 = \frac{n^4 + 3n^3 + n^2 + 3n}{2(n-1)^2 (n^3 + 3n^2 + 2n + 1)} \tag{8.17}$$

$$e_5 = \frac{n^9 - 4n^7 + 24n^6 + 79n^5 + 56n^4 + 14n^3 - 12n^2 - 10n - 4}{2(n-1)^2 (n+1)^2 (n^3 + 3n^2 + 2n + 1)^2} \tag{8.18}$$

It is clear that $c_d < 0$. Let us now study the signs of b_d and a_d.

Sign of b_d

$$b_d \geq 0 \Leftrightarrow \kappa \geq \frac{e_5}{e_4}, \text{ where } \kappa = \frac{X^0_{-1} + X^1_{-1}}{\bar{X}} \tag{8.19}$$

Further, from equations (8.17)-(8.18), $\forall n \geq 2$, $\frac{e_5}{e_4} = \frac{n^9-4n^7+24n^6+79n^5+56n^4+14n^3-12n^2-10n-4}{(n+1)^2(n^3+3n^2+2n+1)(n^4+3n^3+n^2+3n)}$ and $\frac{e_5}{e_4} \leq 1$.

Sign of a_d

Using the variable $\kappa = \frac{X^0_{-1}+X^1_{-1}}{\bar{X}}$, I rewrite equation (8.14) as:

$$a_d = \bar{X}^2 \left[e_1 - e_2\kappa^2 + e_3\kappa\right]$$

For $n = 2$, $e_1 < 0$, $e_2 > 0$, $e_3 > 0$. When $n > 2$, all coefficients are strictly positive. Thus,

- If $n = 2$, there are two positive roots, $\kappa_1 = \frac{e_3-\sqrt{\delta}}{2e_2}$ and $\kappa_2 = \frac{e_3+\sqrt{\delta}}{2e_2}$, where $\delta = e_3^2+4e_2e_1$.
- If $n > 2$, there is a positive and a negative roots, with $\kappa_1 < 0$ and $\kappa_2 > 0$.

Hence, $a_d > 0 \Leftrightarrow$

- $\kappa \in]\kappa_1, \kappa_2[$, if $n = 2$
- $\kappa \in]0, \kappa_2[$, if $n > 2$.

Discriminant

The discriminant of equation (8.13) is:

$$\begin{aligned} \Delta_d &= R^2 \left[r_1 \left(X^0_{-1} + X^1_{-1}\right)^2 + r_2\bar{X}\left(X^0_{-1} + X^1_{-1}\right) + r_3\bar{X}^2\right] \\ \text{i.e., } \Delta_d &= R^2\bar{X}^2 \left[r_1\kappa^2 + r_2\kappa + r_3\right] \end{aligned} \tag{8.20}$$

with $r_1 = e_4^2 - 4e_6e_2$, $r_2 = 4e_6e_3 - 2e_5e_4$, $r_3 = e_5^2 + 4e_6e_1$. $\forall n \geq 2$, $r_1 > 0$, and $r_2 > 0$. Further, $r_3 < 0$ for $n = 2$ and $r_3 > 0$ for $n > 2$.[27]

[27] For the sake of brevity, I did not reproduce the analytical expression of the coefficients r_i. I check the signs numerically for $n = 2$ to $n = 150$.

Hence if $n = 2$, the equation $r_1\kappa^2 + r_2\kappa + r_3$ has two solutions:

$$\begin{aligned}\kappa_1^d &= \frac{-r_2 + \sqrt{\Delta_d}}{2r_1} \approx 0.1\\ \kappa_2^d &= \frac{-r_2 - \sqrt{\Delta_d}}{2r_1} < 0, \text{where } \Delta_d = r_2^2 - 4r_1r_3\end{aligned}$$

If $n > 2$, then all coefficients r_i being strictly positive, $\Delta_D > 0$ for any κ. Hence,

- If $n = 2$, then $\Delta_d < 0$ for $\kappa \in \left[0, \kappa_1^d\right[$. If $\kappa > \kappa_1^d \approx 0.1$, then $\Delta_d > 0$.
- If $n > 2$, then $\Delta_d > 0$.

Equilibrium

The equilibrium is determined by the sign of equation (8.13) and the parameter restrictions β_F and $\bar{\beta}_d$, given by equations (8.5) and (8.7), respectively.

When $\Delta_d > 0$, equation (8.13) has two real roots given by

$$\underline{\beta}_d = \frac{\sqrt{\Delta_d} - b_d}{2a_d} \tag{8.21}$$

$$\underline{\beta}_{d,2} = -\frac{b_d + \sqrt{\Delta_d}}{2a_d} \tag{8.22}$$

It is easy to see that if $a_d > 0$, $\beta_2 < 0$, and if $a_d < 0$, $\beta_2 > \underline{\beta}_d > 0$. Using $\kappa = \frac{X^0_{-1} + X^1_{-1}}{\bar{X}}$, equations (8.21) and (8.22) and (8.14)-(8.16), the roots can be rewritten as:

$$\underline{\beta}_d = \frac{|R|}{\bar{X}} \frac{Z_\kappa^{\frac{1}{2}} - (e_4\kappa - e_5)}{2\left(e_1 - e_2\kappa^2 + e_3\kappa\right)} \tag{8.23}$$

$$\beta_2 = -\frac{|R|}{\bar{X}} \frac{Z_\kappa^{\frac{1}{2}} + (e_4\kappa - e5)}{2\left(e_1 - e_2\kappa^2 + e_3\kappa\right)} \tag{8.24}$$

where $Z_\kappa = r_1\kappa^2 + r_2\kappa + r_3$.

I first study the sign of equation (8.13) independently of the parameter restrictions.

If $n > 2$, $\Delta_d > 0$, hence the equation has two real roots. From the signs of a_d and b_d, there are two thresholds for κ in this case: κ_2 and $\frac{e_5}{e_4}$. Since for all $n \geq 2$, $\kappa_2 \geq 1$ and $\frac{e_5}{e_4} < 1$, it is clear that $\kappa_2 > \frac{e_5}{e_4}$. Then the sign of equation (8.13) is as follows:

- If $\kappa \in \left[0, \frac{e_5}{e_4}\right[$, $a_d > 0$, $b_d < 0$, $c_d < 0$, hence $\beta_2 < 0$, $\underline{\beta}_d > 0$ and $a_d\beta^2 + b_d\beta + c_d \geq 0 \Leftrightarrow \beta > \underline{\beta}_d$
- If $\left[\frac{e_5}{e_4}, \kappa_2\right[$, $a_d > 0$, $b_d > 0$, $c_d < 0$, then $\beta_2 < 0$, $\underline{\beta}_d > 0$ and $a_d\beta^2+b_d\beta+c_d \geq 0 \Leftrightarrow \beta > \underline{\beta}_d$.
- If $\kappa > \kappa_2$, then $a_d < 0$, $b_d < 0$, and $c_d < 0$ and $a_d\beta^2 + b_d\beta + c_d \geq 0 \Leftrightarrow \beta \in \left[\underline{\beta}_d, \underline{\beta}_{d,2}\right[$

When $n = 2$, there are four thresholds κ_1^d, κ_1, $\frac{e_5}{e_4}$ and κ_2, in increasing order. For $\kappa \geq \frac{e_5}{e_4}$, the analysis is similar to the case where $n > 2$. For $\kappa < \frac{e_5}{e_4}$, the intervals are as follows:

- If $\kappa \in \left[0, \kappa_1^d\right[$, $a_d < 0$, $b_d < 0$, $c_d < 0$, and $\Delta_d < 0$, hence $a_d\beta^2 + b_d\beta + c_d < 0$ and there is no predatory trading equilibrium.
- If $\kappa \in \left[\kappa_1^d, \kappa_1\right[$, then $\Delta_d > 0$, but since $a_d < 0$, $b_d < 0$, $c_d < 0$, there are two negative roots, and therefore, there is no predatory trading equilibrium. This case can be grouped with the previous one.
- If $\kappa \in \left[\kappa_1, \frac{e_5}{e_4}\right[$, then $a_d > 0$, $b_d < 0$, $c_d < 0$ and $\Delta_d > 0$. Then $\beta_2 < 0$, $\underline{\beta}_d > 0$ and $a_d\beta^2 + b_d\beta + c_d \geq 0 \Leftrightarrow \beta > \underline{\beta}_d$. Thus this case can be grouped with the one in which $\kappa > \frac{e_5}{e_4}$.

$\Rightarrow$ The $n = 2$ case is thus the same as the $n > 2$ case, except for $\kappa < \kappa_1$.

I now determine the intervals of the predatory trading equilibrium, taking into account the parameter restrictions β_F and $\bar{\beta}_d$, given by equations (8.5) and (8.21), respectively.

<u>*Position of β_F relative to $\bar{\beta}_d$*</u>

From equations (8.5) and (8.21):

$$\bar{\beta}_d > \beta_F \Leftrightarrow a \geq m_1\theta + m_2 \tag{8.25}$$

with $m_1 = \frac{n(n+1)^2}{n^4+4n^3-n^2+4}$ and $m_2 = \frac{n^4+3n^3+n^2+n+2}{n^4+4n^3-n^2+4}$

Note that $m_2 = 1$ when $n = 2$ and $m_2 < 1$ when $n > 2$.
$\Rightarrow$ If $\theta = 0$ (i.e. $X_{-1}^0 = 0$), $\bar{\beta}_d > \beta_F \Leftrightarrow a \geq m_2$, which is always true since $a \geq 1$.
$\Rightarrow$ Proposition 2 follows from this remark and the analysis below.

<u>*Intervals of the predatory trading equilibrium*</u>

The analysis of equation (8.13) gives necessary and sufficient conditions in terms of the variable κ, whereas the parameter restrictions for β_F and $\bar{\beta}_d$ are expressed in terms of θ. Noting that[28]:

$$\kappa = \frac{\theta + 1}{a} \tag{8.26}$$

I rewrite all the conditions in terms of a and θ.

The thresholds in terms of κ are κ_1 (for $n = 2$ only), $\frac{e_5}{e_4}$ and κ_2. Hence using equation (8.26), the corresponding thresholds in terms of a are, in increasing order, $\frac{1}{\kappa_1}\theta + \frac{1}{\kappa_1}$, $\frac{e_4}{e_5}\theta + \frac{e_4}{e_5}$ and $\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}$.

I now compare these thresholds to the condition (8.25). For all $n \geq 2$, $\frac{e_4}{e_5} > m_2 > m_1$, $\frac{1}{\kappa_1} > m_2 > m_1$. Therefore, $\forall n \geq 2$,

$$\begin{cases} \frac{e_4}{e_5}\theta + \frac{e_4}{e_5} > m_1\theta + m_2 \\ \frac{1}{\kappa_1}\theta + \frac{1}{\kappa_1} > m_1\theta + m_2 \end{cases}$$

Further, $\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2} > m_1\theta + m_2$ is equivalent to

$$\theta > \theta^* = \frac{m_2 - \frac{1}{\kappa_2}}{\frac{1}{\kappa_2} - m_1}$$

Since $\forall n \geq 2$, $m_2 > \frac{1}{\kappa_2} > m_1$, $\theta^* > 0$. Hence, combining the equilibrium conditions and the parameter restrictions yields, $\forall n > 2$

- If $a \geq \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then $I_P = \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$
- If $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then $I_P = \left[\underline{\beta}_d \wedge \bar{\beta}_d, \underline{\beta}_{d,2} \wedge \bar{\beta}_d\right[$
- If $\min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right) < a < \max\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, then
  - If $\theta > \theta^*$, then $I_P = \left[\underline{\beta}_d \wedge \beta_F, \underline{\beta}_{d,2} \wedge \beta_F\right[$,
  - If $\theta \leq \theta^*$, then $I_P = \left[\underline{\beta}_d \wedge \bar{\beta}_d, \bar{\beta}_d\right[$.

[28] Using the definition of κ (8.19) and the following notations: $\theta = \frac{X^0_{-1}}{X^1_{-1}}$, $a = \frac{\bar{X}}{X^1_{-1}}$

If $n = 2$, there is an additional case: if $a \geq \frac{1}{\kappa_1}\theta + \frac{1}{\kappa_1}$, there is no predatory trading equilibrium.

In the second case, $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, it is possible to refine the boundaries of the interval I_P and show that it is non-empty, thereby proving the existence of the equilibrium in this case.

Existence conditions

I first show that $\underline{\beta}_d < \bar{\beta}_d$. This case is interesting for $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, hence the interval I consider is $\kappa > \kappa_2$. Using (8.23) and (8.7), and rearranging terms, I get

$$\underline{\beta}_d - \bar{\beta}_d = \frac{|R|}{\bar{X}} \frac{g_2(\kappa)}{(\rho_{0,n-1}\kappa - d_n)(e_1 - e_2\kappa^2 + e_3\kappa)} \tag{8.27}$$

$$\text{with } g_2(\kappa) = (\rho_{0,n-1}\kappa - d_n) Z_\kappa^{\frac{1}{2}} + B_1\kappa^2 + B_2\kappa - B_3 \tag{8.28}$$

where $\forall n \geq 2$, $B_1 = 2e_2 - \rho_{0,n-1}e_4 < 0$, $B_2 = e_5\rho_{0,n-1} + d_n e_4 - 2e_3 < 0$, $B_3 = 2e_1 + d_n e_5 > 0$.[29]

The denominator of equation (8.27) is negative when $\kappa > \kappa_2$, thus $\underline{\beta}_d - \bar{\beta}_d < 0$ iff $g_2(\kappa) \geq 0$. To determine the sign of g_2, I first study its first derivative:

$$g_2'(\kappa) = \rho_{0,n-1} Z_\kappa^{\frac{1}{2}} + (\rho_{0,n-1}\kappa - d_n)\frac{Z_\kappa'}{Z_\kappa^{\frac{1}{2}}} + 2B_1\kappa + B_2$$

The first term of the derivative is positive for any $\kappa > 0$. The second term is also positive, because $\forall n \geq 2$, $\frac{d_n}{\rho_{0,n-1}} < \kappa_2$ and $Z_\kappa' = 2r_1\kappa + r_2 > 0$ for any $\kappa > \kappa_2 > 0$ (r_1 and r_2 being positive for any $n \geq 2$). The third term, however, is negative, because B_1 and B_2 are negative. I will show that $\forall \kappa > \kappa_2$, $g_2'(\kappa) > 0$. To show this, it is enough to show that $\rho_{0,n-1} Z_\kappa^{\frac{1}{2}} + 2B_1\kappa + B_2 \geq 0$.

Since $Z_\kappa = r_1\kappa^2 + r_2\kappa + r_3$ (see equation (8.23)), the following holds for any $\kappa > \kappa_2$:

$$Z_\kappa \geq r_1\kappa^2 + r_2\kappa_2 + r_3$$

and therefore $\rho_{0,n-1}\sqrt{Z_\kappa} \geq \rho_{0,n-1}\sqrt{r_1\kappa + r_2\kappa_2 + r_3}$, which implies that

$$\rho_{0,n-1}\sqrt{Z_\kappa} + 2B_1\kappa + B_2 \geq \rho_{0,n-1}\sqrt{r_1\kappa^2 + r_2\kappa_2 + r_3} + 2B_1\kappa + B_2$$

[29]For the remainder of the proof, I rely again on calculations for the coefficients which are functions of n.

Given that $\forall n \geq 2$, $\rho_{0,n-1}\sqrt{r_1} \geq -2B_1$, the function on the RHS of the inequality is increasing in κ. Hence for $\kappa > \kappa_2$, $\rho_{0,n-1}\sqrt{Z_\kappa}+2B_1\kappa+B_2 > \rho_{0,n-1}\sqrt{r_1\kappa_2^2 + r_2\kappa_2 + r_3}+2B_1\kappa_2+B_2$. The right-hand side of the inequality is positive for all $n \geq 2$, hence $\forall \kappa > \kappa_2$, $\forall n \geq 2$, $g_2'(\kappa) > 0$ and g_2 is increasing on this interval. As a result, one can minor this function by $g_2(\kappa_2)$, with $\forall n \geq 2$, $g_2(\kappa_2) > 0$.

Hence $\forall \kappa > \kappa_2$, $\underline{\beta}_d < \bar{\beta}_d$.

Using a similar reasoning, one can show that $\underline{\beta}_{d,2} > \bar{\beta}_d$ when $\kappa > \kappa_2$. From equations (8.24) and (8.7), $\underline{\beta}_{d,2} < \bar{\beta}_d$ is equivalent to $h_2(\kappa) > 0$, with

$$h_2(\kappa) = -\left(\rho_{0,n-1} - d_n\right)\sqrt{Z_\kappa} + B_1\kappa^2 + B_2\kappa - B_3$$

The function $-\left(\rho_{0,n-1} - d_n\right)\sqrt{Z_\kappa}$ is always negative, as well as $B_1\kappa^2 + B_2\kappa - B_3$. Thus $\forall \kappa > \kappa_2$, $\underline{\beta}_{d,2} > \bar{\beta}_d$. ■

Corollary 7

Proof Suppose that $a \leq \min\left(\frac{1}{\kappa_2}\theta + \frac{1}{\kappa_2}, m_1\theta + m_2\right)$, and consider $p(\kappa) = 1 - \hat{q}(\kappa) = \frac{\underline{\beta}_d}{\bar{\beta}_d}$. From equations (8.23) and (8.7), we can write

$$p(\kappa) = \frac{\left(\rho_{0,n-1}\kappa - d_n\right)\left(Z_\kappa^{\frac{1}{2}} - \left(e_4\kappa - e_5\right)\right)}{2\left(e_1 - e_2\kappa^2 + e_3\kappa\right)}$$

Hence the first derivative w.r.t. κ, after regrouping terms, is

$$p'(\kappa) = \frac{\left(e_1 - e_2\kappa^2 + e_3\kappa\right)\left(\rho_0 Z_\kappa + \left(\rho_0 - d_n\right)\left(2r_1\kappa + r_2\right)\right) - \left(e_3 - 2e_2\kappa\right)\left(\rho_0\kappa - d_n\right)2Z_\kappa}{2Z_\kappa^{\frac{1}{2}}}$$
$$+\left(e_5\rho_0 + e_4d_n - 2e_4\rho_0\kappa\right)\left(e_1 - e_2\kappa^2 + e_3\kappa\right) + \left(2e_2\kappa - e_3\right)\left(-e_4\rho_0\kappa^2\left(e_5\rho_0 + e_4d_n\right)\kappa - d_ne_5\right)$$

It is enough to show that p is increasing when $\kappa \geq \kappa_2$. I start by developing and rearranging terms of the numerator in the first line. Using that $Z_\kappa = r_1\kappa^2 + r_2\kappa + r_3$, I get after a few calculations that the numerator is equal to $H_1\kappa^4 + H_2\kappa^3 + H_3\kappa^2 + H_4\kappa + H_5$, with

$$\begin{aligned}
H_1 &= e_2r_1\rho_0;\ H_2 = 2e_2\rho_0r_2 - 2r_1d_2e_2 + r_1e_3\rho_0 \\
H_3 &= 3r_3e_2\rho_0 - 2r_2d_ne_2 + 3r_1\rho_0e_1 + e_3\rho_0r_2;\ H_4 = 2r_2\rho_0e_1 - r_3e_3\rho_0 - 4r_3d_ne_2 - e_1r_1d_n \\
H_5 &= 2r_1e_3d_n - e_1r_2d_n
\end{aligned}$$

Now consider the second line in p' and rearrange terms. This gives: $H_6\kappa^2 - H_7\kappa + H_8$, with

$$H_6 = e_2\left(e_5\rho_0 + e_4 d_n\right) + e_3 e_4 \rho_0;\ H_7 = 2e_4 e_1 \rho_0 + 2e_2 d_n e_5;\ H_8 = e_1\left(e_5\rho_0 + e_4 d_n\right) + d_n e_5 e_3$$

Hence the sign of p' is the same as the sign of

$$\phi_\kappa = H_1\kappa^4 + H_2\kappa^3 + H_3\kappa^2 + H_4\kappa + H_5 + 2Z_\kappa^{\frac{1}{2}}\left(H_6\kappa^2 - H_7\kappa + H_8\right)$$

Calculating the coefficients H_i, which are functions of n, we find that H_1, H_2, H_3, H_6 and H_8 are positive for any $n \geq 2$. However, for $n \geq 2$, H_4 is negative, H_7 is positive and H_5 becomes negative for $n \geq 4$. Given the signs of the coefficients, to show that p' is positive for $\kappa \geq \kappa_2$, it is enough to show $H_3\kappa^2 + H_4\kappa + H_5 \geq 0$ and $H_6\kappa^2 - H_7\kappa + H_8$ on this interval.

First, consider $H_3\kappa^2 + H_4\kappa + H_5 \geq 0$. Since $H_3 > 0$, it is increasing for $\kappa \geq -\frac{H_4}{2H_3}$, which calculations show is smaller than κ_2. Further, I find that for any $n \geq 2$, $H_3\left(\kappa_2\right)^2 + H_4\kappa_2 + H_5 > 0$. Next, consider $H_6\kappa^2 - H_7\kappa + H_8$ and apply the same steps. H_6 is positive and the function peaks in $\frac{H_7}{2H_6}$, which I find is smaller than κ_2 for $n \geq 2$. Further, I find that $H_6\kappa^2 - H_7\kappa + H_8 > 0$. As a result, p' is positive for $\kappa \geq \kappa_2$, hence $\hat{q}$ is decreasing on its interval.

■

Corollary 8

Proof I start with $\theta > 0$:

$$\begin{aligned} Q^d \geq 0 \quad &\Leftrightarrow \quad X^0_{-1} + X^1_{-1} + \frac{n}{n+1}\left(\frac{R}{\beta} - \bar{X}\right) \geq 0 \\ &\Leftrightarrow \quad X^0_{-1} + X^1_{-1} - \frac{n}{n+1} a X^1_{-1} \geq \frac{n}{n+1}\frac{|R|}{\beta},\ \text{using } \bar{X} = aX^1_{-1} \end{aligned}$$

With a small enough, $X^0_{-1} + \left(1 - \frac{n}{n+1}a\right)X^1_{-1} > 0$. Hence,

$$Q^d \geq 0 \ \Leftrightarrow \ \beta \geq \beta^h \equiv \frac{n}{n+1}\frac{|R|}{X^0_{-1} + \left(1 - \frac{n}{n+1}a\right)X^1_{-1}}$$

If $\theta = 0$, we need to prove that $\beta^h \geq \beta_F$. Using the expression for β_F from Lemma 6, we get:

$$\beta^h \geq \beta_F \ \Leftrightarrow \ \frac{n+1}{n} - a \leq \frac{n+2}{n}a - \frac{n+1}{n} \ \Leftrightarrow \ \frac{2\left(n+1\right)}{n} \leq \frac{2\left(n+1\right)}{n}a$$

Since $a \geq 1$, this inequality is always satisfied. ■

Corollary 9

Proof Using Proposition 5, we get:

$$E_0(p_1 - p_0) = D - \bar{p}_0 - \frac{\beta}{n+1}\bar{X} - \frac{|R|}{n+1} = \frac{n|R|}{n+1} - \frac{\beta}{n+1}\bar{X}$$

Thus $E_0(p_1 - p_0) \geq 0 \Leftrightarrow n|R| - \beta\bar{X} > 0 \Leftrightarrow \beta < \frac{n|R|}{\bar{X}}$. Since$\beta < \beta_F = \frac{n|R|}{(n+2)\bar{X} - (n+1)X_{-1}^1}$, and $\beta_F \leq \frac{n|R|}{\bar{X}} \Leftrightarrow X_{-1}^1 \leq \bar{X}$, we have $E_0(p_1) \geq p_0$. Clearly, $E_0(p_1 - p_0)$ increases with —R— and decreases with X_{-1}^1.

The illiquidity discount at time 1, $\Gamma_1 = -\frac{\beta\bar{X} + |R|}{n+1}$. Hence Γ_1 is decreasing in $\bar{X}$ and $|R|$. ■

9 Additional derivations for the no trading case

Proposition 1

Proof From Proposition 5, the driver of the equilibrium is the position of a relative to $\max\left(m_2, \frac{1}{\kappa_2}\right)$ and $\min\left(m_2, \frac{1}{\kappa_2}\right)$.

If $X_{-1}^0 = 0$, then $\theta = 0$, and the equilibrium condition simplifies as follows:

- Since $\forall n \geq 2$, $m_2 > \frac{1}{\kappa_2}$ and since $\frac{1}{\kappa_2} \leq 1 \leq a$, the case $a < \min\left(m_2, \frac{1}{\kappa_2}\right)$ does not exist.
- Further, $\forall n \geq 2, m_2 \geq 1$, hence the case $\min\left(m_2, \frac{1}{\kappa_2}\right) < a < \max\left(m_2, \frac{1}{\kappa_2}\right)$ does not exist either.

The only remaining case is thus $a \geq \max\left(m_2, \frac{1}{\kappa_2}\right) = m_2$. Since $\frac{1}{\kappa_2} < m_2 \leq 1$ for all $n \geq 2$, the condition on a is always satisfied. Hence if $\theta = 0$, the equilibrium condition for the equilibrium with predatory trading is $\beta \in \left[\underline{\beta}_d \wedge \beta_F, \beta_F\right[$. ■

Proposition 3

Proof The equilibrium with distress occurs on a non-empty interval iff $\underline{\beta}_d < \beta_F$. Using equations (8.23) and (8.5):

$$\begin{aligned} \underline{\beta}_d - \beta_F &= \frac{|R|}{\bar{X}} f(n,a) \\ \text{with } f(n,a) &= \frac{(u_1 - u_{2a})\left(\sqrt{\gamma_{3a}} - \gamma_{5a}\right) - 2\gamma_{6a}}{2\gamma_{6a}(u_1 - u_{2a})} \end{aligned} \tag{9.1}$$

Similarly, using equations (8.23) and (7.16), I get:

$$\begin{aligned} \underline{\beta}_d - \underline{\beta}_{nd} &= \frac{|R|}{\bar{X}} g(n,a) \\ \text{with } g(n,a) &= \frac{\lambda_2\left(\sqrt{\gamma_{3a}} - \gamma_{5a}\right) - a\gamma_{6a}\left(\sqrt{A_2} - \lambda_5\right)}{2\gamma_{6a}\lambda_2} \end{aligned} \tag{9.2}$$

The no-trading and predatory trading equilibria coexist iff $g(n,a) > 0$. ■

Lemma 7

Proof We can recover Q^d from equation (3.12): $Q^d = \frac{n}{n+1}\frac{R}{\beta} - \frac{n}{n+1}\bar{X} + X^1_{-1}$. Using p_0^{nd} from Lemma 2, $p_0^{nd}\left(Q^{nd}, \bar{X}\right) = \bar{p_0} \Leftrightarrow Q^{nd} = \frac{n+1}{n+2}\frac{R}{\beta} - \bar{X} + X^1_{-1}$. Thus

$$Q^{nd} \geq Q^d \Leftrightarrow \frac{1}{(n+1)(n+2)}\frac{R}{\beta} \geq \frac{1}{n+1}\bar{X}$$

The left-hand side is strictly negative, while the right-hand side is strictly positive. Hence $Q^{nd} < Q^d$. Further, note that since $\bar{X} > X^1_{-1}$, $Q^{nd} < 0$.

To understand this impact of the change in price schedule on the equilibrium conditions, I redo the analysis of Lemma 6 based on the no-distress price schedule, following identical steps. The prey's problem is

$$\max_{x_0^1, x_0^1 \leq \bar{X} - X^1_{-1}} B^1_{-1} - x_0^1\left[\bar{p_0} - \beta\frac{n+2}{n+1}\left(\bar{X} - X^1_{-1} - x_0^1\right)\right] + X_0^1\left[D - \frac{\beta}{n+1}\left(\bar{X} - \frac{R}{\beta}\right)\right]$$

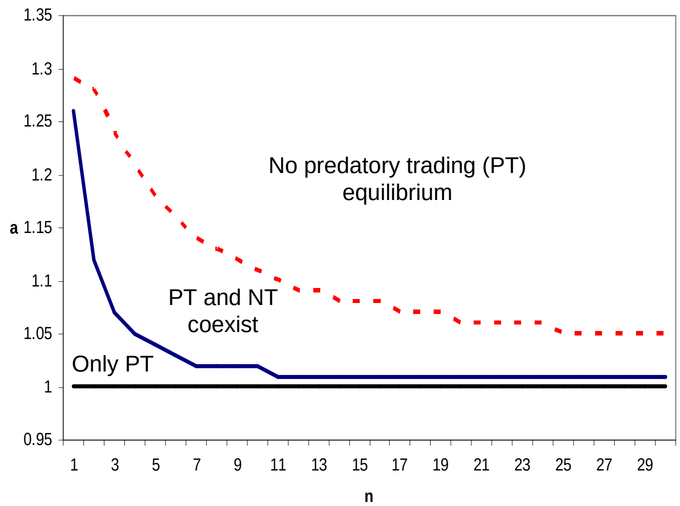
I write the Lagrangian of the problem and solve for the zero of the first-order condition (assuming the Lagrangian multiplier is 0). I get:

$$x_0^1 = \frac{n}{2(n+2)}\frac{|R|}{\beta} + \frac{1}{2}\left(\frac{n+1}{n+2}\bar{X} - X^1_{-1}\right)$$

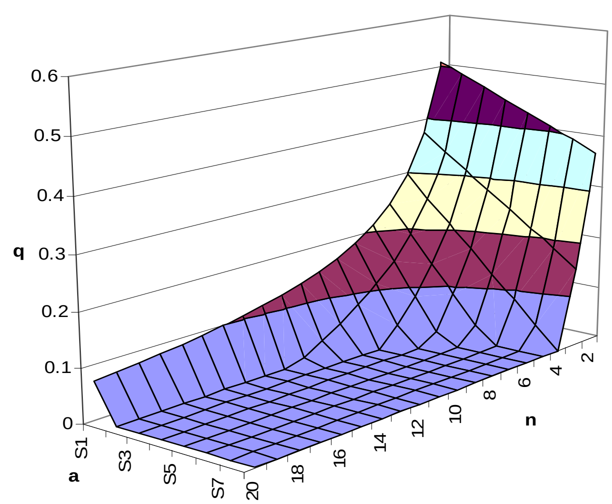
Hence the constraint on the prey's position is not binding if $\frac{n}{2(n+2)}\frac{|R|}{\beta} + \frac{1}{2}\left(\frac{n+1}{n+2}\bar{X} - X^1_{-1}\right) \leq$

$\bar{X} - X_{-1}^1$, which is equivalent to $\beta < \tilde{\beta}_F \equiv \frac{|R|}{\frac{n+3}{n}\bar{X} - \frac{n+2}{n}X_{-1}^1}$. In the proof of Lemma 6, I show that $\beta_F = \frac{|R|}{\frac{n+2}{n}\bar{X} - \frac{n+1}{n}X_{-1}^1}$, hence $\beta_F > \tilde{\beta}_F$.

Similarly, one can predict how the condition for ruling out self-fulfilling distress would change. Since predators have less price impact when the price schedule is p_0^{nd}, it will harder, conditional on distress, to trigger it, thus there should be a larger interval on which predatory trading is not self-fulfilling. In other words, $\tilde{\bar{\beta}}^d > \bar{\beta}^d$. ■

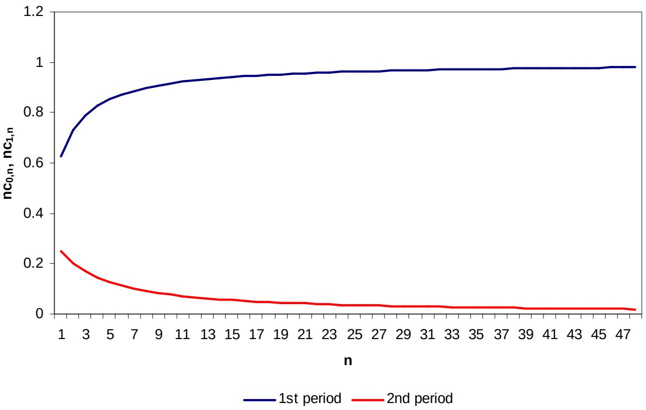


(a) Equilibria with or without distress

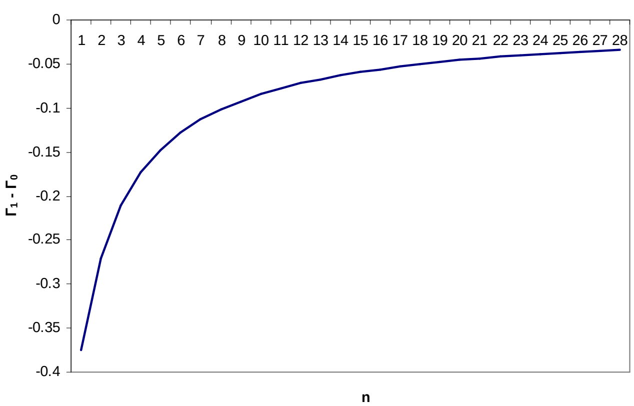


(b) "Net" probability of predatory trading

Figure 1: Coexistence of equilibria and "net" probability of predatory trading as a function of the number of predators n, and the prey's leverage capacity, $a = \frac{\bar{X}}{X^1_{-1}}$. In Panel (b), a varies from 1 (S1) to 1.07 (S7). The calculations assume that β is uniformly distributed between 0 and β_F.



(a) Equilibrium trades



(b) Convergence

Figure 2: Equilibrium trades and speed of convergence of the price towards the fundamental value of the asset as a function of the number of strategic traders in the no-distress equilibrium.